Minimum Energy Channel Codes for Nanoscale Wireless Communications

Murat Kocaoglu, *Student Member, IEEE*, and Ozgur B. Akan, *Senior Member, IEEE*

***Abstract*—It is essential to develop energy-efficient communication techniques for nanoscale wireless communications. In this paper, a new modulation and a novel minimum energy coding scheme (MEC) are proposed to achieve energy efficiency in wireless nanosensor networks (WNSNs). Unlike existing studies, MEC maintains the desired code distance to provide reliability, while minimizing energy. It is analytically shown that, with MEC, codewords can be decoded perfectly for large code distances, if the source set cardinality is less than the inverse of the symbol error probability. Performance evaluations show that MEC outperforms popular codes such as Hamming, Reed-Solomon and Golay in the average codeword energy sense.**

***Index Terms*—CNT antennas, minimum energy coding, THz channel, nanosensors, nanoscale wireless communications.**

I. INTRODUCTION

WIRELESS nanosensor networks (WNSNs), which are collections of nanosensors with communication capabilities, are believed to have revolutionary effects on our daily lives [1]. The development of novel communication techniques suitable for nanodevice characteristics is essential for WNSNs.

One of the most promising building blocks for future nanodevices are carbon nanotubes (CNT). CNTs are rolled up graphene sheets with nano dimensions that can be used as nanoantennas, nano sensing units and nanobatteries [2], [3]. The resonant frequency of CNT antennas lies in the Terahertz band of the spectrum (0.1-10 THz). This band is not utilized by macro applications and is a candidate for communications between nanodevices [1]. The main challenge of using the THz band is the absorption of EM waves by water vapour molecules, which makes communication impractical by causing severe path loss and molecular noise [4].

Potential nanosensors have significantly different performance metrics than the macro sensors. Although no complete nanonode has yet been implemented, it is anticipated that power and energy efficiency are of the most critical measures due to their extremely small size. Hence, developing novel energy-efficient communication techniques is essential.

Manuscript received December 9, 2011; revised May 8 and October 8, 2012; accepted January 12, 2013. The associate editor coordinating the review of this paper and approving it for publication was Z. Wang.

A preliminary version of this work was presented in IEEE INFOCOM'12 [6].

This work was supported in part by the Turkish Scientific and Technical Research Council under grant #109E257, by the Turkish National Academy of Sciences Distinguished Young Scientist Award Program (TUBA-GEBIP), by IBM through IBM Faculty Award, and by Turk Telekom under Grant Number 11315-04.

The authors are with the Department of Electrical and Electronics Engineering, Koc University, Istanbul, Turkey (e-mail: {mkocaoglu, akan}@ku.edu.tr).

Digital Object Identifier 10.1109/TWC.2013.022113.2190

Employing channel coding at the nanoscale is critical to assure reliable communication between nanodevices. The classical channel codes have various design considerations such as the efficient use of code space, as in perfect codes, bounded decoding complexity as the Shannon capacity is approached, as in Turbo or LDPC codes, or low encoding and decoding complexity as in cyclic and convolutional codes. However, the coding scheme for nano wireless communications should consider the *energy dissipation at the transmitter* as the main metric, since nanonodes run on a strict energy budget. Thus, classical codes are not suitable. Unlike most of the classical codes, minimum energy coding minimizes the average codeword energy, if OOK is the underlying modulation [5]. However, the existing minimum energy codes are unreliable.

To address these needs, we develop a novel minimum energy channel code (MEC), that is reliable and suitable for nano communications. Proposed code provides the minimum average codeword energy of all the block codes, given that OOK is used as the modulation scheme. With OOK, average codeword energy is the symbol energy times average codeword weight; therefore, average energy is minimized by minimizing the average code weight. For this, codeword weights and sourceword-codeword mappings are chosen such that the expected code weight is minimized at the cost of increased codeword length, hence increased delay. Lengthy codewords could increase the energy dissipation at the transmitter due to energy dissipation of the nanosensor circuitry. This implies a tradeoff between the transmission and processing energies and a discrete optimization problem could arise. However, such an analysis is not feasible today, since it is inaccurate to estimate the energy dissipation at the nano processing units, as no complete nanonode architecture is yet available. The suitability of MEC for nanoscale communications is shown by obtaining the achievable rate at the nanonode.

In this paper, we significantly extend our preliminary work in [6] and [7]. We propose an OOK-based multi-carrier modulation suitable for WNSNs. Carriers are chosen to exploit the absorption characteristics of the THz channel. To address the low complexity requirement at the nanosensor nodes, low-complexity medium access techniques are investigated. Moreover, we develop four lemmas and the proofs of the Theorems presented in [6]. Performance evaluations are extended to cover energy per information bit comparisons with popular codes. Additionally, we analyze the effect of interference in cell-based WNSNs. Micro nodes act as central controller units of each cell to enable inter-cell communication and intra-cell coordination. The maximum number of quantization levels and the effects of cell coverage ratio are investigated.

The remainder of this paper is organized as follows: In Section II, the existing work on WNSNs and minimum energy codes are presented. In Section III, low-complexity medium access techniques and WNSN architecture are discussed. We develop MEC in Section IV and derive the relevant analytical expressions in Section V. In Section VI, MEC performance is compared with popular block codes. Moreover, effects of cell radius and coverage ratio on the maximum number of source quantization levels in a cell-based WNSN using MEC are investigated. In Section VII, concluding remarks are given.

II. Related Work

WNSNs can be used for sensing and data collection with extremely high resolution and low power consumption in various applications [1]. In [3], the authors introduce CNT sensor networks and present major challenges to be addressed for their realization. The authors in [1] provide a detailed survey on the state-of-the-art in nanosensors and emphasize potential applications and design challenges. In [4], the THz channel absorption and noise characteristics and capacity are investigated. Despite these studies, channel coding in nano wireless communications is still a barren field. Recently, using low-weight codes with femtosecond-long OOK pulses is proposed in [8] to mitigate interference in nanonetworks. However, to the best of our knowledge, the need for developing channel codes to achieve energy-efficient and reliable nano communications has not been addressed so far.

The idea of using low-weight channel codes together with OOK modulation to reduce energy consumption is first proposed in [5] for sensor networks. Choosing codewords for each source outcome such that mean codeword energy is less than any other choice of codeword mappings is called minimum energy coding. The authors show that, for a given codebook, sorting codewords in increasing code weight order and assigning source symbols in decreasing probability order, such that the most probable source symbol is mapped to the codeword with the smallest weight yields the optimum average code weight. Later, the authors in [9] propose using codewords with maximum weight of 1. Such a mapping corresponds to minimum energy coding, if the all-zero codeword is mapped to the most probable source outcome. However, this code is not reliable since its code distance is 1, and any bit error pattern is uncorrectable. Therefore, development of reliable minimum energy codes has been an open issue.

In this paper, first, we present a new modulation scheme suitable for nano wireless communications in the THz band. Contrary to the existing nanoscale communication schemes in which the whole THz band is utilized, our scheme alleviates the need to deal with the performance degradation due to molecular absorption lines and molecular noise. Later, we address the need for reliable minimum energy codes and develop such codes that have controllable reliability via code distance. Lastly, we show the suitability of MEC for nanosensors by investigating the achievable information rate and interference limited source set cardinality in WNSNs with MEC.

III. Wireless NanoSensor Network Architecture

Energy-efficiency and suitability for the THz channel are the prior concerns for the realization of WNSNs. Complexity of the nanosensor must also be kept as low as possible. In this section, we explain the communication techniques we develop for nanosensors and discuss a feasible extension to WNSNs.

The main functionalities of the nanonode structure shown in Fig. 1 can be found in [3]. We propose using multiple CNT antennas to utilize a number of available frequency windows in THz band. Required energy can be provided by the battery via nano energy-harvesting systems [10]. Sensing is also CNT-based. Nanosensor readings are quantized to M levels. No source coding is employed so as not to increase complexity. Each source signal level is mapped to $length - n$ channel codewords with a combinatorial nano-circuit. Realization of such a processing is not clear today. However, studies on CNT-based logic gate applications [11] increase hope. The processing block is also responsible for carrier generation. Even though carrier generation in nano domain is not clear, it is shown that, with their unique properties such as slowing down surface EM waves, CNTs can also be used to generate THz waves much easier than the classical techniques [12]. Control block contains a separate antenna for the control of the nanonode from a central unit. Nanonode activates and transmits only when this antenna is excited. This functionality is required for low complexity multiple access in WNSNs.

A. Multi-carrier OOK Modulation

Motivated with the THz channel characteristics, we propose a *multi-carrier modulation scheme for nanoscale wireless communications*. Each codeword is transmitted in parallel over different carriers. Our frequency choice considers carriers' suitability for transmission in the THz channel. As previously mentioned, the THz channel consists of several frequency windows with low absorption and low molecular noise, termed as *available windows*, which depends on the transmission distance and water vapour amount on the transmission path [4]. Carrier frequencies are chosen among these windows in the THz channel. CNTs are used as nanoantennas to radiate each carrier, as shown in Fig. 1. Each frequency window is utilized separately. Bandwidth increase is prohibited by the molecular absorption lines. Decreasing the bandwidth results in increased energy consumption per symbol, since symbol duration increases. Hence, we select bandwidth as the same as the width of the available frequency windows. Hence, picoseconds long sinusoidal pulses are used, which span a frequency band of 100-200 GHz, corresponding to the width of most of the windows in the THz channel [4].

Channel codes with minimum average weight are utilized, together with OOK modulation at each carrier to reduce the energy consumption. Proposed coding achieves the minimum codeword energy and guarantees a minimum Hamming distance at the price of lengthy codewords. Multi-carrier modulation mitigates delays due to lengthy codewords of MEC in WNSN node. The number of multi-carrier signals can be chosen to satisfy a certain delay requirement.

B. WNSN Cell Architecture

We consider a cell-based WNSN for the first time in the literature. A cell is composed of a micro node, and nanosensor nodes scattered around it. In order to reduce the interference,

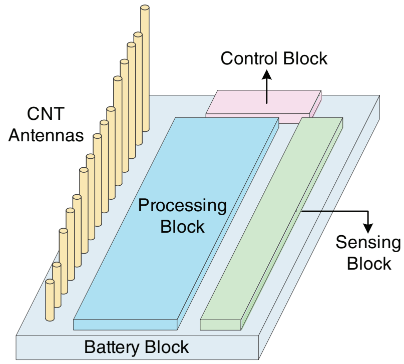


Fig. 1. Proposed nanosensor node architecture.

nanonodes are deployed within a radius of αR, where R is the cell radius and α is called *the coverage ratio* satisfying $0 < \alpha \leq 1$. To keep the complexity of the nanonodes low, all the control and scheduling issues are left to the micro node within the cells. A nanonode starts transmission only when an activation signal is sent by the micro node. As suggested in [13], kHz band can be used for this activation signal, with vibrating CNTs. The central micro node provides not only control, but also synchronization among the nanosensors. It is assumed that the micro node is capable of receiving the THz waves. In the current literature, many studies on CNT based THz receivers demonstrated that CNT bundles can be used for efficient THz detection at room temperature [14]. With their employment, multi-wavelength THz receivers with micro dimensions will be available in the near future.

Let N be the number of nodes in a WNSN cell and l the number of channels for multi-carrier modulation. Assume that all the nanonodes are within a range to directly communicate with the micro node. There are two reliable medium access techniques, keeping complexity at the micro node:

Single Control Signal: Nanonodes start transmission simultaneously through disjoint sets of channels (frequencies). To keep complexity at the micro node, different sets of frequencies must be used by each nanonode, and a common synchronization signal must be broadcast from the micro node for signalling the transmission. Nl different THz frequency windows, and a single kHz band are allocated to a single cell. This is an FDMA-based scheme, as separate frequency windows are allocated to each nanosensor node.

Multiple Control Signals: Nanonodes use the same set of frequencies for transmission. The micro node uses control signals at different frequencies for each nanonode sequentially, as nanonodes utilize the same THz channels. Allocation of l THz and N kHz bands are needed. This is similar to TDMA, since all the nodes use the channel in different time intervals.

In the following, we assume that the micro node uses multiple control signals, since the number of frequency windows in the THz channel is limited and demodulating a number of different THz signals significantly increases complexity.

IV. Minimum Energy Channel Coding

We propose new channel codes, which minimize the average code weight. Such codes are equivalent to the codes minimizing average codeword energy for the systems employing OOK modulation. This is because, no energy is dissipated when 0 symbol is transmitted and no ARQ scheme is employed in nano communications for retransmissions.

For block codes, a codebook is defined as any selection of fixed length codewords, mapped to source symbols. For unique decodability, this mapping should be one-to-one. *Weight* is the number of non-zero entries in the codeword. As we deal with binary codes, weight is equivalent to the number of $1s$ in the codeword. Weight enumerator of a code is the polynomial $W_{\mathbb{C}}(z) = \sum_i c_i z^i$, where c_i is the number of codewords with weight i. Additionally, the distance (or Hamming distance) between two codewords is defined as the number of bits in which they differ. Code distance is the minimum of the distances between all codewords. In minimum distance decoding, which is the presumed decoding strategy, the received n-tuple is mapped to the closest codeword. Codes with distance d can correct $\lfloor \frac{d-1}{2} \rfloor$ errors, and reliability increases with distance, since more error patterns can be corrected.

Codewords with lower weight results in less energy dissipation, when transmission of 0 symbol requires less energy than the transmission of 1 symbol. OOK is an example of such modulation schemes, in which transmission of 0s require no energy. OOK is also favorable at nanoscale due to its simplicity. As pointed out, there has been a need to develop reliable minimum energy codes. To address this issue, we develop minimum energy channel codes with any code distance d to guarantee reliability. Proposed code minimizes the expected codeword weight, depending on the source probability distribution. In this section, we derive MEC and obtain the corresponding minimum average code weight.

In the nanonodes, each codeword has the same probability of occurrence as the source outcomes that they are mapped to, since no source coding mechanism is employed. This brings a new problem into the picture: *What is the codebook selection that minimizes the average code weight for any input probability distribution?* This problem can be interpreted as finding the weight enumerator and mapping between codewords and sourcewords such that the expected codeword weight for a given input probability mass function is minimized. It is trivial that for no code distance constraint, i.e., $d = 1$, assigning codewords with maximum weight of 1 minimizes the average weight, as proposed in [9]. To obtain an analytical solution, we modify the minimum energy code problem such that codeword length n is kept unconstrained. Later, we develop the required code length for different cases in Section V.

Let M, d, p_{max}, X represent number of codewords, code distance, maximum probability in any discrete distribution and the source random variable, respectively.

Lemma 1. *For any finite M, there exists a finite n_0 such that a constant weight code $\mathbb{C}$ of length-n_0 containing the codeword c can be constructed with code distance d, if and only if $weight(c) \geq \lceil \frac{d}{2} \rceil$:*

$$\exists \mathbb{C} : dist(\mathbb{C}) \geq d \text{ for } c \in \mathbb{C} \Leftrightarrow \textit{weight(c)} \geq \lceil d/2 \rceil .$$

Lemma 2. *Any codebook with code distance of d contains at most a single codeword with weight less than $\lceil d/2 \rceil$.*

Lemma 3. *Any two codeword c_i and c_j of a code with distance d should satisfy the inequality $weight(c_i)$ +*

$weight(c_j) \geq d$.

Let $\mathbb{C}_i$ be the code with weight enumerator

$$W_{\mathbb{C}_i}(z) = z^{\lfloor \frac{d}{2} \rfloor - i} + (M-1) z^{\lceil \frac{d}{2} \rceil + i}. \quad (1)$$

The code $\mathbb{C}_i$ contains a single codeword with weight $\lfloor \frac{d}{2} \rfloor - i$ and all the other codewords have weight $\lceil \frac{d}{2} \rceil + i$. Let codeword with weight $\lfloor \frac{d}{2} \rfloor - i$ be assigned to the source symbol with maximum probability, i.e., p_{max}. Let $E_{\mathbb{C}_i}$ represent expected code weight for code $\mathbb{C}_i$.

Lemma 4. $E_{\mathbb{C}_{i+k}} < E_{\mathbb{C}_i}$ *if* $p_{max} > 0.5, \forall k > 0$.

Proof: Let β represent $\lfloor \frac{d}{2} \rfloor$. Then

$$\begin{aligned} E_{\mathbb{C}_i} &= p_{max}(\beta - i) + (1 - p_{max})(d - \beta + i) \\ &= p_{max}(2\beta - 2i - d) + d - \beta + i. \\ \Rightarrow E_{\mathbb{C}_i} - E_{\mathbb{C}_{i+k}} &= k(2p_{max} - 1). \end{aligned}$$

Hence, since k is positive, $E_{\mathbb{C}_{i+k}} < E_{\mathbb{C}_i}$ if $p_{max} > 0.5$. ■

Theorem 1. *Let* $X = x_i$ *has probability* $p_i \in \{p_1, p_2, ..., p_M\}$ *and* p_{max} *be* $\max(p_i)$. *For a desired code distance* d, *the minimum expected codeword weight,* $E(w)$ *is*

$$\min(E(w)) = \begin{cases} (1 - p_{max})d, & p_{max} > \frac{1}{2}, \\ \frac{d}{2}, & p_{max} < \frac{1}{2},\ d \textit{ even} \\ \lceil \frac{d}{2} \rceil - p_{max}, & p_{max} < \frac{1}{2},\ d \textit{ odd} \end{cases}, \quad (2)$$

Proof: Let c_i be the codeword assigned to the source symbol x_i that has probability p_i and w_i represent $weight(c_i)$.

From Lemma 1, we know that, a $weight - \lceil \frac{d}{2} \rceil$ code can be constructed with finite code length for any M. Therefore, $\min(E(w)) \leq \lceil \frac{d}{2} \rceil$. From Lemma 2, we know that we can decrease the weight of only a single codeword below $\lceil \frac{d}{2} \rceil$. Then the bound can safely be improved by switching the code weight of the most probable outcome to $\lfloor \frac{d}{2} \rfloor$, since the resultant code will still satisfy the distance condition. This leads to a bound valid for any source probability distribution:

$$\min(E(w)) \leq p_{max} \lfloor d/2 \rfloor + (1 - p_{max})(d - \lfloor d/2 \rfloor) \quad (3)$$

From Lemma 3, to further reduce the weight of the most probable codeword, we should increase the weight of all the other codewords to satisfy $weight(c_i) + weight(c_j) = d$ for any i, j. Lemma 4 shows that this operation, i.e., increasing i in (1), decreases the average weight, if $p_{max} > 0.5$. Hence, minimum average weight is obtained when $i = \lfloor \frac{d}{2} \rfloor$ for $p_{max} > 0.5$. This yields the code $\mathbb{C}_d$ with the enumerator $W_{\mathbb{C}_d}(z) = z^0 + (M-1)z^d$, giving the average weight

$$E(w) = (1 - p_{max})d. \quad (4)$$

Note that by mapping the codeword of weight $\lfloor \frac{d}{2} \rfloor$ to any symbol with probability $p < p_{max}$, this bound cannot be reached, since decreasing the weight of this chosen codeword does not decrease the average weight as $p < p_{max}$ forces $p < 0.5$. Furthermore, no other weight decreasing scheme can be applied after such an assignment. Hence, the best bound is obtained by decreasing the weight of the most probable codeword, which is given in (4). If p_{max} is less than 0.5, expected weight cannot be decreased more than (3) by Lemma 4. After simple manipulations, (2) can be easily obtained. ■

Another problem definition is as follows: *What is the minimum expected code weight for code distance* d *and maximum weight* k*?* k is the maximum high symbols in a codeword that the nanonode can supply power for. If $k < \lceil d/2 \rceil$, there is no way to satisfy code distance. Hence, we assume $k \geq \lceil d/2 \rceil$.

Theorem 2. *Let* $X = x_i$ *has probability* $p_i \in \{p_1, p_2, ..., p_M\}$ *and* p_{max} *be* $\max(p_i)$. *For a desired code distance* d *and maximum codeword weight* k, *if* $\lceil d/2 \rceil \leq k < d$ *is satisfied, minimum expected codeword weight,* $E(w)$ *is given by*

$$\min(E(w)) = \begin{cases} p_{max}(d - 2k) + k, & p_{max} > \frac{1}{2}, \\ \frac{d}{2}, & p_{max} < \frac{1}{2},\ d \textit{ even} \\ \lceil \frac{d}{2} \rceil - p_{max}, & p_{max} < \frac{1}{2},\ d \textit{ odd} \end{cases}, \quad (5)$$

Proof: It is clear that, if $p_{max} < 0.5$, bound given in Theorem 1 can be achieved, since $k \geq \lceil d/2 \rceil$. However, if $p_{max} > 0.5$, by Lemma 4, i in (1) should be increased to reduce the average code weight, and could at most be $i = k - \lceil \frac{d}{2} \rceil$ due to weight constraint. Hence, for $\lceil d/2 \rceil \leq k < d$,

$$\min(E(w)) = p_{max}(d - k) + (1 - p_{max})k.$$

Combining both cases, theorem is obtained in few steps. ■

Note that if the maximum allowable codeword weight is greater than or equal to d, Theorem 2 reduces to Theorem 1, showing that Theorem 2 is a generalization of Theorem 1.

Another point is that, if we use all zero codeword in the codebook (the case when $p_{max} > 0.5$ and $k \geq d$), we cannot distinguish if the transmitter sent data or remained silent, as both yield the same output. For this, we can put a minimum distance of d with silence case also for all the codewords. This forces us to choose $weight - d$ codewords for all the symbols to minimize expected code weight, resulting in an average codeword weight of d. However, as explained in Section III, since a micro node provides synchronization, we assume that all zero codeword can be distinguished from the silence.

Note that MEC only determines the weight enumerator, not the codebook. Hence, minimum energy codes are not unique, since multiple codebooks satisfy the MEC weight enumerator.

V. Analytical Results and MEC Parameters

Power dissipated for codeword i is $P_i = w_i P_{sym}$, where P_{sym} is the symbol power. Then the average power is

$$E(P) = \sum_{i=1}^{M} w_i p_i P_{sym} = E(w) P_{sym}. \quad (6)$$

(6) also shows the average power per $\log(M)$ bits, since codewords carry $\log(M)$ bits of information. For different source distributions, information per codeword will be different from an information theoretic point of view. However, for simplicity, we assume each codeword carries $\log(M)$ bits of information, leaving the information theoretic analysis to a future study.

We have developed MEC by keeping the codeword length unconstrained. Let us investigate the minimum length of MEC.

A. Minimum Codeword Length

n_{min} is the minimum codeword length required to satisfy the MEC weight enumerator for given M and d. n_{min} is important as it yields the minimum delay due to transmission of codewords. $A(n, d, w)$ is the maximum number of codewords of length n with code distance d and fixed code weight w.

1. *$p_{max} < 0.5$, d even:* Weight enumerator of MEC is $W_{\mathbb{C}}(z) = Mz^{d/2}$. Therefore, $n_{min} = \min\{n : A(n, d, d/2) \geq M\}$. Since $1s$ in each codeword are disjoint, $n_{min} = \frac{Md}{2}$.

2. *$p_{max} < 0.5$, d odd:* From Theorem 1, we know that the weight enumerator is $W_{\mathbb{C}}(z) = z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}$. $1s$ in all the codewords should be disjoint with the $1s$ in the most probable codeword, i.e., the codeword with weight $\lfloor \frac{d}{2} \rfloor$. Hence, $n_{min} = \lfloor \frac{d}{2} \rfloor + \min\{\tilde{n} : A(\tilde{n}, 2m+1, m+1) \geq M-1\}$, where $d = 2m+1$. The following property is helpful [15]:

$$\begin{aligned} &A(n, 2m-1, w) = A(n, 2m, w) \\ \Rightarrow &A(\tilde{n}, 2m+1, m+1) = A(\tilde{n}, 2m+2, m+1). \end{aligned} \tag{7}$$

Therefore, $\min\{\tilde{n}\} = (m+1)(M-1)$. Hence, $n_{min} = m + (m+1)(M-1) = \lceil \frac{d}{2} \rceil M - 1$.

3. *$p_{max} > 0.5$:* In this case, MEC has the weight enumerator $W_{\mathbb{C}}(z) = z^0 + (M-1)z^d$ and maps the all-zero codeword to the most probable source event. Minimum codeword length is found as $n_{min} = \min\{n : A(n, d, d) \geq M-1\}$. In the literature, there is no explicit formulation for $A(n, d, d)$. We can use the existing lower bounds on the code size. From [15],

$$\begin{aligned} &A(n, 2m, w) = A(n, 2m-1, w) \geq \frac{1}{q^{m-1}}\binom{n}{w} \\ \Rightarrow &A(n, d, d) \geq \frac{1}{q^{\lceil \frac{d}{2} \rceil - 1}}\binom{n}{d}, \end{aligned} \tag{8}$$

where q is a prime power such that $q \geq n$.

The codewords for $p_{max} < 0.5$ and $d-even$ case can be constructed by cyclic shifting of a $d/2-length$ block of $1s$ by an amount of $d/2$. Based on this cyclic shifting idea, we have developed a code construction scheme. In this approach, blocks of $1s$ are shifted by proper amounts to satisfy the Hamming distance with the previous codeword. The obtained minimum codeword length under such a construction is

$$n_{min} = d + (M-2)\lceil d/2 \rceil. \tag{9}$$

Sample codebooks generated by this scheme can be found in the Appendix. This construction achieves the minimum code length for $p_{max} < 0.5$ and d–$even$ since $1s$ should be disjoint. Unexpectedly, this scheme also achieves minimum codeword length for $p_{max} < 0.5$ and $d-odd$, since (9) reduces to n_{min} obtained for this case. However, the codeword length of this scheme is significantly greater than the minimum codeword length for $p_{max} > 0.5$. For example, for $M = 112$ and $d = 8$, minimum code length of 27 is sufficient from (8), instead of $n = 448$, obtained from (9). However, to be able to numerically analyze the error performance, and obtain results valid for all the p_{max} and d values, we use (9) in our analysis.

If the minimum Hamming distance between the codewords is increased, more codeword errors can be corrected. However, the codeword length of MEC also increases with the code distance, which result in a larger number of error patterns. Thus, increasing code distance does not necessarily increase reliability of MEC. Hence, analysis of error correcting capability of MEC for large code distance is worth considering.

B. Error Resilience

The received n-tuples are mapped to the codeword to which they are closest in terms of Hamming distance. Then the probability that codeword is correctly decoded is

$$\xi_d = \sum_{i=0}^{\lfloor \frac{d-1}{2} \rfloor} \binom{n_{min}}{i} p_s^i (1-p_s)^{n_{min}-i}. \tag{10}$$

We have shown that for sufficiently large distance, codewords are correctly decoded with high probability, if the symbol error probability is less than the inverse of source set cardinality.

$$\xi = \lim_{d \to \infty} \xi_d = \begin{cases} 1, & p_s < 1/M \\ 0, & p_s > 1/M \end{cases}. \tag{11}$$

(11) is proven in Appendix A. Hence, perfect communication can be achieved among nanosensor nodes and micro node, if $M < 1/p_s$, by keeping the code distance sufficiently large. Hence, if symbol error probability is decreased, nanosensor readings can be quantized with smaller quantization steps.

The micro node utilizes coherent detection and hard decoding to detect the transmitted symbol. Therefore, symbol error probability is given as $p_s = 0.5\left[1 - erf\left((A^2/8\sigma_n^2)^{0.5}\right)\right]$, where A is the received signal level when symbol 1 is transmitted, and σ_n^2 is the noise power at the receiver. Here, we assume that the transmitter and receiver nanonodes are stationary, and path loss is constant. Therefore, the received power for the high symbol is constant, shown by A. It is sufficient to consider the spreading loss only, since carrier frequencies are at the available frequency windows in the THz band, where molecular absorption is low. Interference created by other cells due to frequency reuse should be considered in the noise power calculation. Let S be the set of nodes interfering with node i. Then the signal and noise powers are

$$P_r = \frac{P_{sym}}{A(f, r)} = \frac{A^2}{2}, \; \sigma_n^2 = k_B T B + P_{sym} \sum_{i \epsilon S} \frac{1}{A(f_i, r_i)}, \tag{12}$$

where k_B, T, B, r are Boltzmann constant, temperature, bandwidth and transmission distance. $A(f, r) = (4\pi fr/c)^2$ is the loss term, where f is frequency and c is the speed of light.

C. Energy per Information Bit

Next, we obtain energy per information bit to demonstrate the energy efficiency of our coding scheme. Probability that a codeword is correctly decoded, which is obtained in (10), can also be obtained as follows using law of large numbers:

$$\xi_d \approx \frac{\text{\# of codewords correctly decoded}}{\text{\# of codewords transmitted}} \tag{13}$$

for a large number of transmitted codewords, for a code with distance d. Hence, if Q codewords are transmitted, then $\log(M)Q\xi_d$ bits of information is received. Average energy transmitted per codeword is $E_C = P_{sym}E(w)T_{sym}$ joules, where T_{sym} is the symbol duration. Then, the total energy

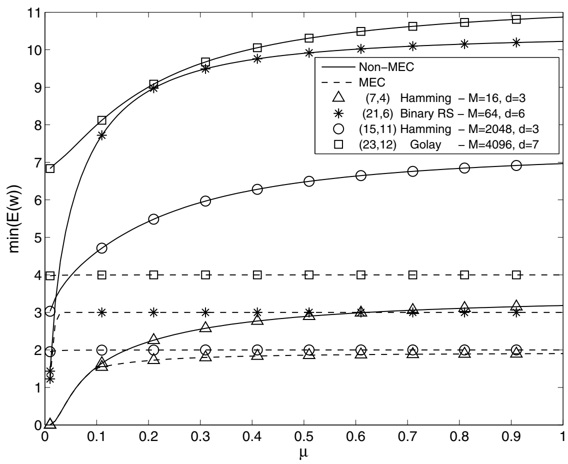


Fig. 2. Minimum code weight vs. source mean for (7,4), (15,11) Hamming, (21,6) Binary Reed-Solomon and (23,12) Golay code and corresponding MEC

dissipated for Q transmissions is $E_C Q$. Therefore, the average energy per bit is expressed as the ratio $E_C Q/\log(M)Q\xi_d$, i.e.,

$$\eta = \frac{E(w)P_{sym}T_{sym}}{\log(M)\xi_d} \text{ joules/bit}. \quad (14)$$

D. Spectral Efficiency

Finally, we investigate spectral efficiency, which is one of the important parameters in a communication system. It is defined as the ratio of data rate to the bandwidth and yields how efficiently channel bandwidth is utilized. Information transmitted per codeword per second is given by $\xi_d \log M/nT_{sym}$. Bandwidth required per codeword in Hz is given by lB. Then spectral efficiency of MEC is obtained as

$$\nu = \frac{\xi_d \log(M)}{nlT_{sym}B} \approx \frac{\xi_d \log(M)}{2nl} \text{ bps/Hz}. \quad (15)$$

VI. Performance Evaluation and Discussions

In this section, we investigate error correction capability and energy-efficiency of MEC via numerical evaluations of analytical parameters in MATLAB. An (n,k) code maps 2^k sourcewords into $length-n$ codewords. For comparison, we use MEC with $M = 2^k$. MEC is compared with the (7,4), (15,11) Hamming, (21,6) binary Reed-Solomon and (23,12) Golay codes. The Hamming codes are distance-3 codes, and can correct 1 bit errors whereas the Golay code is distance-7 and can correct 3 bit errors. The minimum distance of (21,6) binary Reed-Solomon code is known to be 6.

A. Performance of Minimum Energy Coding

1) Average code weight vs. source distribution: MEC is compared with the classical block codes in Fig. 2 in terms of expected code weight. To minimize code weight for the Hamming, Reed-Solomon and Golay codes, more probable source symbols are assigned to codewords with less weight, using the corresponding weight enumerators. We use the binary expansion of 8-ary (7,2) Reed Solomon code for which a sample weight enumerator is given in [16]. We use normalized samples of an exponential pdf with varying mean - μ in a fixed interval to generate the discrete distributions with different variances. It is clear from Fig. 2 that MEC is superior, i.e., classical codes are not as efficient in terms of average energy per codeword. Performance gap between the codes closes as μ, i.e., variance of discrete distribution is decreased, which increases p_{max}. This is expected, since all the codes contain the all-zero codeword, which mainly determines minimum average weight for large p_{max}. As observed, if μ exceeds a threshold, entering $p_{max} < 0.5$ region, MEC clearly outperforms the other codes due to the abrupt change of its weight distribution.

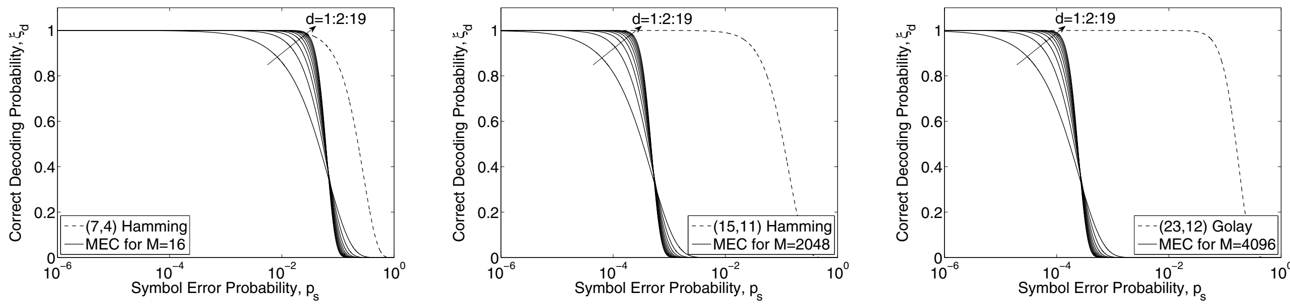
2) Correct codeword decoding vs. symbol error: Codeword decoding performances of MEC, Golay and Hamming codes are illustrated in Fig. 3(a)-3(c). MEC is not as effective as the others in terms of error correction. This is due to the different codeword lengths. Lengthy codes have more uncorrectable error patterns, which decreases the error correction probability. As observed in Fig. 3, correct decoding probability increases with code distance and approaches to 1, if symbol error probability, p_s, is less than the inverse of source set cardinality, $1/M$, verifying (11). Intuitively, transmitted information increases with M, which requires more reliable channels.

3) Energy efficiency vs. symbol error: The average energy per received bit, i.e., η as given in (14), is shown in Fig. 4(a)-4(c) for a symbol energy of 10^{-5} pJ, which is justified in Section VI-B. Samples of a Gaussian distribution with $\sigma = 0.5$ are taken and normalized. η is calculated for each case separately using (14). MEC is better in terms of average energy per bit for symbol error probabilities less than a threshold. As p_s exceeds the threshold, average energy per bit exponentially increases, since correct codeword decoding is unlikely. Note that the observed behavior is dominated by $1/\xi$ factor in (14).

B. Achievable Rate of WNSN Nodes

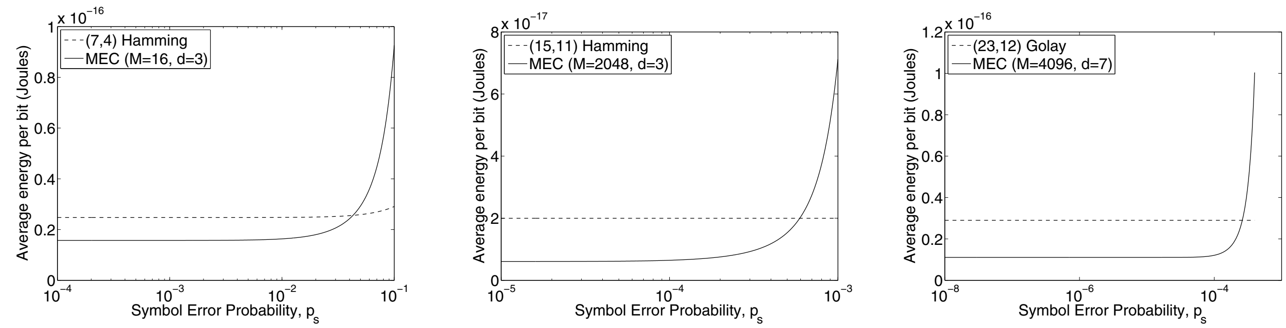
In this section, we investigate the feasibility of MEC for WNSN nodes, using state-of-the-art power and energy limits in the nano-domain. It is theoretically calculated in [17] that a CNT antenna can radiate EM waves with power up to $5\mu W$. We allocate the available power equally to each CNT antenna. In [18], an ultra-nano capacitor to store energy obtained from piezoelectric nano-generator energy harvesting system is investigated. Up to 800 pJ of energy can be stored in the capacitor. Charging time for the capacitor depends on the frequency of vibration that the nanonode is exposed to. To charge nano capacitor with 100 pJ of energy, 160 cycles are required. If nodes gather energy from a 50 Hz source, such as a vent, 160 cycles correspond to 3.2 seconds to charge the battery up to $\varepsilon_{battery} = 100$ pJ. T_{sym}, i.e., symbol duration is 10 picoseconds due to the proposed modulation. Hence, for l number of carriers, symbol energy is constant and equals to $\varepsilon_{sym} = P_{sym}T_{sym} = \frac{1}{0.2l} \times 10^{-5}$ pJ.

Therefore, using the energy limits of nanobattery and energy per symbol, a nanosensor node can transmit $\varepsilon_{battery}/\varepsilon_{sym} = 0.2l \times 10^7$ high symbols in 3.2 seconds. We can calculate the achievable transmission rate as the amount of information that



(a) (b) (c)

Fig. 3. Codeword decoding probability at the receiver for (7,4), (15,11) Hamming and (23,12) Golay codes and MEC with odd distances from 1 to 19.



(a) (b) (c)

Fig. 4. Average energy per bit comparison between (7,4), (15,11) Hamming codes, (23,12) Golay code and MEC.

can be carried with these high symbols. Let $p_{max} < 0.5$ and d be even for simplicity. Then, using (2) and (6), $\log(M)$ bits of information is carried with codewords of weight $d/2$ on the average. Hence, average transmission rate is limited by

$$R = \frac{0.4l \times 10^7 \log(M)}{3.2d} = 1.25l \times \frac{\log(M)}{d}\text{Mbps}. \quad (16)$$

Therefore, transmission rate can be increased with increased l, M and decreased d. Note that for transmission rate to be equal to the information rate, channel should be sufficiently reliable. Decreasing the code distance is not really an option to increase the rate, if it decreases reliability. Also, increasing the number of carriers decreases energy per symbol, making each symbol harder to decode. Hence, to maximize the information rate, parameters should be adjusted considering the reliability of the channel. As an example, energy-limited rate for $M = 16$, $d = 5$ and $l = 4$ is 4 Mbps. We can say for favorable channel conditions a nanonode can achieve information rate of 4 Mbps with the current nano energy harvesting systems.

Note that it takes n/l symbol times to transmit a codeword. This sets another limit on the transmission rate, i.e.,

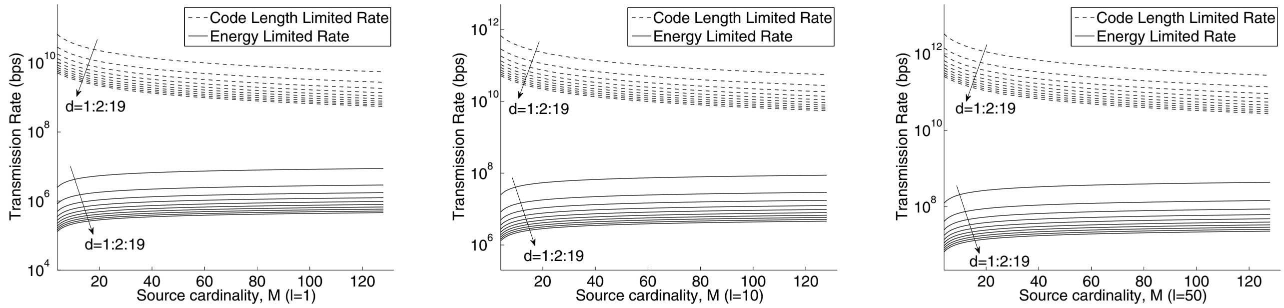
$$R < \frac{\log(M)l}{n_{min}T} = \frac{\log(M)l}{(d + (M-2)\lceil d/2 \rceil)T}. \quad (17)$$

In (17), $T_{sym} = 10$ psec. is the symbol duration. This bound is illustrated in Fig. 5(a), 5(b) and 5(c) for $l = 1$, $l = 10$ and $l = 50$, respectively. Fig. 5 shows that energy budget currently available at the nanonode limits its rate, rather than the codeword length. As a result, codeword length, which is the major drawback of MEC, does not limit the transmission rate. Moreover, since rate is limited by the available energy, MEC provides the maximum information rate compared to other block codes, as it minimizes the codeword energy. As observed in Fig. 5, code length allows rates up to $10s$ of Gbps.

C. Effect of Interference on WNSN Node Quantization

We assume a frequency reuse ratio of 1/4 and a large network. As explained, a TDMA-based scheme, in which channel use times are allocated by the central micro node, is assumed within each cell. As a result, at most one nanonode transmits at any time instant inside a cell, mitigating the intra-cell interference. Due to this, interference from the other cells using the same frequencies is only due to a single nanonode, which is active at the time of transmission. This leads to an analysis, independent from either the size or the nanonode density of the network. The effects of 50 closest cells using the same set of frequency bands are considered, which is sufficient, since interference power is inversely proportional with the distance square. Noise is the thermal noise as in (12), however, its effect is negligible compared to the interference. Channel loss is spreading loss only, since available frequency windows with low absorption are utilized. We assume $l = 5$ with frequencies 0.1, 0.3, 1, 1.5 and 2 THz. All the channels contribute to average symbol error probability equally.

We consider the worst case scenario and assume that the nanonode transmitting to the micronode is at the edge of the cell, i.e., the transmission distance is at its maximum; and the interfering nanonodes in other cells are deployed as close to our node as possible, maximizing the interference. Cell coverage ratio of α is utilized to reduce interference, i.e., the nanonodes are deployed within a range of αR, where R is



(a) (b) (c)

Fig. 5. Transmission rate of MEC for different values of carriers, i.e., $l = 1$, $l = 10$ and $l = 50$.

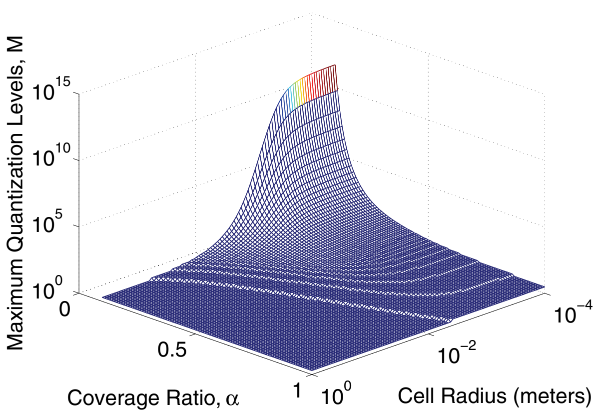


Fig. 6. Maximum number of quantization levels at the nanonode for MEC.

the cell radius. The decoder is assumed to conduct coherent detection of the received signal with hard decoding, since the time instant that nanonode initializes its transmission is declared by the micro node. As indicated, perfect transmission can be achieved with MEC, if $M < 1/p_s$. Hence, maximum source set cardinality M can be obtained from the interference limited symbol error probability, which is shown in Fig. 6. As observed, employing a coverage ratio $\alpha < 1$, a large number of quantization levels can be achieved. More optimistic assumptions, like cooperation between micro nodes, or specific deployment of nodes to reduce the interference, could improve the results. Hence, MEC provides reliability in a large cell-based WNSN with cell radius up to several millimeters.

D. WNSN Performance with Random Nanonode Distribution

We investigate the symbol error probability at a micronode in a WNSN composed of 6 cells. Interference is due to two other cells, since frequency reuse ratio is 1/4. For each α, nanonodes in the cells are randomly deployed in the range αR. We use a single carrier frequency, i.e., $l = 1$, for simplicity, and assume 100 nanonodes in each cell. Symbol error probability depends on the selection of interfering nanonodes.

We evaluate the variation of symbol error probability with α and obtain its maximum, average and minimum for $R = 0.1$ cm, $R = 1$ cm, and $R = 10$ cm in Figs. 7(a), 7(b) and 7(c), respectively. Since error probability is too small for MATLAB to evaluate a non-zero value in the best case, where active node minimizes interference out of 100 nodes, we use linear scale in the y-axis. Hence, we can achieve very small error probabilities by proper positioning and timing of nanonodes, and adjusting α. Also, if cell radius is sufficiently small, large α values might be used, i.e., reliable communication can be achieved without constraints on the nanonode deployment.

VII. Conclusions

In this paper, we propose a multi-carrier OOK modulation, motivated with the THz channel characteristics, and develop a novel minimum energy channel code, MEC, for nano communications in cell-based WNSNs. MEC satisfies a minimum Hamming distance to guarantee reliability. It is analytically shown that codewords can be decoded perfectly using MEC with large code distance, if the number of quantization levels is less than the inverse of symbol error probability. Simulations show that, the proposed code is superior to popular block codes such as Hamming, Reed-Solomon and Golay. The state-of-the-art nanoscale power and energy limits are used to obtain achievable rates of nanonodes, which are expected to be on the order of $Mbps$, neglecting the processing power. Numerical results show that MEC is an energy-efficient and reliable code for future WNSNs with cell radius up to several millimeters.

Appendix A
Proof of Reliability of MEC

Correct decoding probability for large code distance d is

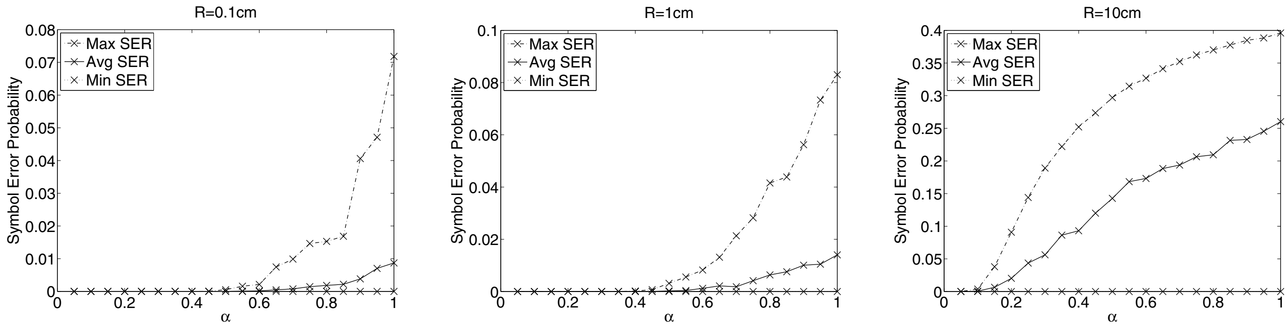
$$\xi = \lim_{d\to\infty} \sum_{i=0}^{\frac{d}{2}-1} \binom{dM/2}{i} p_s^i (1-p_s)^{dM/2-i}$$

$$= \lim_{n\to\infty} \sum_{i=0}^{\frac{n}{M}-1} \binom{n}{i} p_s^i (1-p_s)^{n-i},$$

$$= \lim_{n\to\infty} 0.5 \left(1 - erf\left(\frac{n/M - 1 - np_s}{\sqrt{2np_s(1-p_s)}}\right)\right) \quad (18)$$

$$= \begin{cases} 1, & p_s < 1/M \\ 0, & p_s > 1/M \end{cases}. \quad (19)$$

Expression in (18) is the cumulative distribution function of Gaussian distribution with mean np and variance $np(1-p)$ and erf is the error function. Equality in (18) follows from that, for large n, binomial distribution is approximated by Gaussian.



(a) (b) (c)

Fig. 7. Symbol error rate at micronode vs. α for different R values for uniformly distributed nanonodes.

APPENDIX B
SAMPLE CODEBOOKS

From Theorem 1, MEC has the weight enumerators

$$W_{\mathbb{C}}(z) = \begin{cases} z^0 + (M-1)z^d, & p_{max} > 0.5 \\ z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}, & p_{max} < 0.5. \end{cases}$$

Sample codebooks for $p_{max} < 0.5$ and $p_{max} > 0.5$ with $d = 4$ and $M = 3$ can be respectively generated as

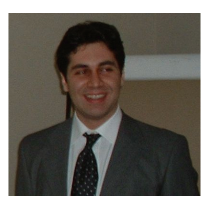
$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Note that each row represents a codeword.

REFERENCES

[1] I. F. Akyildiz and J. M. Jornet, "Electromagnetic wireless nanosensor networks," *Nano Commun. Net.*, vol. 1, pp. 3–19, 2010.
[2] G. W. Hanson, "Fundamental transmitting properties of carbon nanotube antennas," *IEEE Trans. Antennas Propag.*, vol. 53, pp. 3426–3435, 2005.
[3] B. Atakan and O. B. Akan, "Carbon nanotube sensor networks," in *2009 IEEE NanoCom*.
[4] J. M. Jornet and I. F. Akyildiz, "Channel capacity of electromagnetic nanonetworks in the Terahertz band," in *2010 IEEE ICC*.
[5] C. Erin and H. H. Asada, "Energy optimal codes for wireless communications," in *1999 IEEE CDC*.
[6] M. Kocaoglu and O. B. Akan, "Minimum energy coding for wireless nanosensor networks," in *2012 IEEE INFOCOM*.
[7] M. Kocaoglu and D. Malak, "On the node density limits and rate-delay-energy tradeoffs in ad hoc nanonetworks with minimum energy coding," in *2012 IEEE MoNaCom*.
[8] J. M. Jornet and I. F. Akyildiz, "Low-weight channel coding for interference mitigation in electromagnetic nanonetworks in the terahertz band," in *2012 IEEE ICC*.
[9] Y. Prakash and S. K. S. Gupta, "Energy efficient source coding and modulation for wireless applications," in *2003 IEEE WCNC*.
[10] S. Xu *et al.*, "Self-powered nanowire devices," *Nat. Nanotechnol.*, vol. 5, pp. 366–373, 2010.
[11] Y. Huang *et al.*, "Logic gates and computation from assembled nanowire building blocks," *Science*, vol. 294, pp. 1313–1317, 2001.
[12] O. V. Kibis *et al.*, "Generation of terahertz radiation by hot electrons in carbon nanotubes," *Nano Lett.*, vol. 7, pp. 3414–3417, 2007.
[13] I. F. Akyildiz and J. M. Jornet, "The Internet of nano-things," *IEEE Wireless Commun.*, vol. 17, pp. 58–63, 2010.
[14] K. S. Yngvesson *et al.*, "Experimental detection of terahertz radiation in bundles of single wall carbon nanotubes," in *2008 ISSTT*.
[15] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*. North-Holland, 1977.
[16] F. Pollara and S. Arnold, "Binary weight distributions of some Reed-Solomon codes," *TDA Progress Rep.*, vol. 42, 1992.
[17] Y. Wang *et al.*, "Terahertz radiation from armchair carbon nanotube dipole antenna," *Chinese Phys. B*, vol. 18, 2009.
[18] J. M. Jornet and I. F. Akyildiz, "Joint energy harvesting and communication analysis for perpetual wireless nanosensor networks in the terahertz band," *IEEE Trans. Nanotech.*, vol. 11, pp. 570–580, 2012.

Murat Kocaoglu received the B.S. in Electrical and Electronics Engineering with a minor in Physics from Middle East Technical University, Ankara, Turkey in 2010, and M.S. in Electrical and Electronics Engineering at Next-generation and Wireless Communications Laboratory, from Koc University, Istanbul, Turkey in 2012. His research interests include nanoscale communications, energy efficient channel and network coding and information theory.



Ozgur B. Akan (M'00-SM'07) received the Ph.D. in electrical and computer engineering from Georgia Institute of Technology, Atlanta, in 2004. He is currently a Full Professor with the Department of Electrical and Electronics Engineering and the Director of the Next-generation and Wireless Communications Laboratory, Koc University, Istanbul, Turkey. His current research interests include wireless communications, nanoscale and molecular communications, and information theory.