

# Nanoscale Magneto-Inductive Communication

Deniz Kilinc Ozgur B. Akan

Next-generation and Wireless Communications Laboratory (NWCL)  
Department of Electrical and Electronics Engineering  
Koc University, Istanbul, Turkey  
Email: dkilinc, akan@ku.edu.tr

**Abstract**—The nanonetworks constructed by interconnecting nanodevices using wireless communication allow nanodevices to perform more complex functions by means of cooperation between them. For the first time in the literature, we introduce a novel nanoscale communication technique: Nanoscale Magneto-Inductive (NMI) communication. The magnetic coupling between nanocoils establishes a communication channel between them. The electromagnetic (EM) waves at nanoscale encounter two problems: high molecular absorption rates and frequency selective channel characteristics. The novel NMI communication solves these problems by introducing low absorption losses and flat channel characteristics. In the paper, we first present the physical model of the point-to-point NMI communication. Then, we introduce the waveguide technique for the NMI communication. To assess the performance of the point-to-point and the waveguide NMI communication methods, we derive path loss expressions for both methods. The results show that using waveguide technique in the NMI communication significantly reduces the path loss and increases feasible communication range. Based on the numerical performance evaluation, the NMI communication stands as a promising solution to nanoscale communication between nanodevices.

## I. INTRODUCTION

Nanoscale communication between nanodevices is a quite novel and interdisciplinary concept which includes nanotechnology, biotechnology, and communication technology [1]. The nanonetworks constructed by interconnecting nanodevices expands the capabilities of single nanodevices by means of cooperation between them [2]. Several techniques in the literature are presented for the realization of the nanoscale communication namely electromagnetic, acoustic, or molecular communication [3], [4], [5], [6]. However, for the first time in the literature, we introduce a novel nanoscale wireless communication concept: Nanoscale Magneto-Inductive (NMI) communication in which the magnetic coupling between nanocoils is used for wireless communication at the nanoscale.

Using electromagnetic (EM) waves for wireless communication at nanoscale has several disadvantages which are high absorption losses due to molecular absorption and frequency selective characteristics of the channel [7]. The molecular

absorption loss is caused by the process by which part of the transmitted EM wave is converted into internal kinetic energy of some of the molecules in the communication medium [8]. Since different molecule types have different resonant frequencies and the absorption at each resonance spreads over a range of frequencies, the nanoscale EM communication channel is very frequency-selective [7].

The NMI communication stands as a promising alternative method for nanoscale wireless communication because it solves the problems associated with the nanoscale EM communication. Since EM waves are not used, the NMI communication overcomes the high absorption losses because of molecular absorption. Furthermore, in the NMI communication, the channel conditions depend on the magnetic permeability of the medium [11]. Thus, having a communication medium with uniform permeability enables constant channel conditions for the NMI communication. However, the point-to-point NMI communication is a short-range nanoscale communication because the strength of the magnetic field falls off much faster than the EM waves [9], [10]. That is, whereas the molecular absorption in the NMI communication is much less than the EM waves, the path loss of the NMI communication may be higher than the EM communication. However, in the NMI communication, the path loss can be reduced by forming a waveguide structure with passive relay nodes similar to waveguides used in Magneto-Inductive (MI) communication [10].

The MI communication has recently been introduced for wireless underground [10] and underwater communication [11]. Communication with EM waves in these mediums is not feasible due to high attenuation rates. Since the magnetic permeability characteristics of underground and underwater environments are uniform and similar to air [10], the MI communication is a promising method to communicate in dense mediums such as soil and water. Therefore, the NMI communication can be successfully utilized in solid, liquid or gas medium. Furthermore, for nanomedicine applications, the NMI communication can be employed in blood or tissue liquid without having very high attenuation rates. For example, in [12], a MI communication network is used to provide both a communication link between implanted small devices inside the human body and a communication link between an outer device and the implanted devices. Moreover, in the same study, the power for the implanted devices is provided using the MI

The authors are with the Next-generation and Wireless Communications Laboratory (NWCL), Department of Electrical and Electronics Engineering, Koc University, Istanbul, 34450, Turkey (e-mail: {dkilinc, akan}@ku.edu.tr).

This work was supported in part by the Turkish Scientific and Technical Research Council under grant #109E257, by the Turkish National Academy of Sciences Distinguished Young Scientist Award Program (TUBA-GEBIP), and by IBM through IBM Faculty Award.

communication.

In this paper, we first present the model of the point-to-point NMI communication in which a single transmitter and a single receiver nanocoils are used. Since the geometry of planar nanocoils is suitable for the manufacturing processes of integrated circuit production, we employ planar nanocoils in our model. The equivalent circuit of the transmitter and receiver nanocoils are used to derive the analytical expression for the path loss in the NMI communication channel. Using a waveguide structure greatly reduces the path loss and increases the communication range of the MI communication [11]. Then, we introduce the waveguide model for the NMI communication by employing passive relay nanonodes between the transmitter and receiver. The analytical expression for the path loss in the NMI communication waveguide is also obtained.

The rest of the paper is organized as follows. In Section II, we present the physical model of the point-to-point NMI communication channel and underline the governing physical laws and mathematical formulations. In Section III, we introduce the waveguide model of the NMI communication. In Section IV, we discuss the performance evaluations of both the point-to-point and the waveguide NMI communication based on numerical analyses. Finally, Section V concludes the paper.

## II. PHYSICAL MODEL OF POINT-TO-POINT NMI COMMUNICATION

In this section, we present the physical model of the Nanoscale Magneto-Inductive (NMI) communication between a single transmitter nanodevice (TN) and a single receiver nanodevice (RN). In NMI communication, the information transmission and reception is achieved using a planar nanocoil as shown in Fig 1(a). The magnetic coupling between the transmitter and receiver nanocoils establishes the NMI communication channel.

A sinusoidal voltage source is used in TN, i.e.,  $v_T(t) = V_0 \cos(\omega t)$ , where  $\omega$  and  $V_0$  are the angle frequency and amplitude of the voltage source, respectively. The sinusoidal voltage source causes a sinusoidal current to pass through the nanocoil. This current induces another sinusoidal current in the receiver nanocoil and it is used to accomplish the magneto-inductive communication between TN and RN. In Fig 1(c), the transformer model of the NMI communication is shown. The mutual inductance  $M$  between transmitter and receiver nanocoils represents the coupling between these coils.  $L_1$  and  $L_2$  are the self-inductances of the transmitter and receiver nanocoils, respectively.  $R_1$  and  $R_2$  are the resistances, and  $C_1$  and  $C_2$  are the parasitic capacitances of the nanocoils.  $Z_L$  is the load impedance which represents the current consumption of the nanochip included in RN.  $I_1$  is the sinusoidal current passing through the transmitter nanocoil caused by the transmitter voltage source and  $I_2$  is the sinusoidal current induced in the receiver nanocoil by  $I_1$ . The voltage on the load resistor is denoted by  $v_R(t)$ . According to the transformer model illustrated in Fig 1(c), the phasor analysis is given by

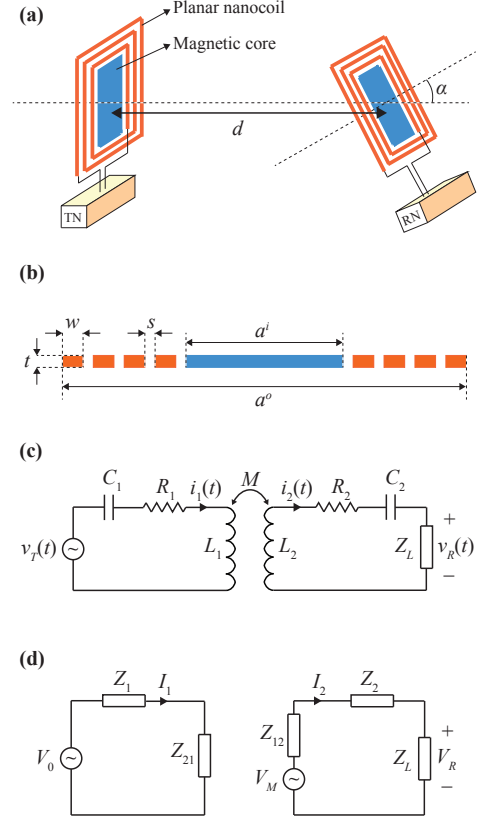


Fig. 1. The NMI communication model and circuit models of the NMI communication. (a) The model of the NMI communication. (b) The side view of a square planar nanocoil. (c) The transformer model of the NMI communication. (d) The equivalent circuit of the transformer model.

$$\begin{aligned} V_0 &= I_1 \left( R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) - j\omega M I_2 \\ 0 &= I_2 \left( R_2 + j\omega L_2 + \frac{1}{j\omega C_2} + Z_L \right) - j\omega M I_1. \end{aligned} \quad (1)$$

Then, the equivalent circuit of the NMI communication model can be derived as shown in Fig 1(d) where

$$\begin{aligned} Z_1 &= R_1 + j\omega L_1 + 1/j\omega C_1 \\ Z_{21} &= \frac{\omega^2 M^2}{Z_L + R_2 + j\omega L_2 + 1/j\omega C_2} \\ Z_2 &= R_2 + j\omega L_2 + 1/j\omega C_2 \\ Z_{12} &= \frac{\omega^2 M^2}{R_1 + j\omega L_1 + 1/j\omega C_1} \\ V_M &= -j\omega M \frac{V_0}{R_1 + j\omega L_1 + 1/j\omega C_1}. \end{aligned} \quad (2)$$

According to the equivalent circuit of the NMI communication model, the transmitted power, denoted by  $P_t$ , and the received power, denoted by  $P_r$ , are given as

$$\begin{aligned} P_t &= \frac{1}{2} \text{Re} \left\{ \frac{|V_0|^2}{Z_1 + Z_{21}} \right\} \\ P_r &= \frac{1}{2} \text{Re} \left\{ \frac{Z_L |V_M|^2}{|Z_2 + Z_{12} + Z_L|^2} \right\} \end{aligned} \quad (3)$$

where  $\text{Re}$  denotes the real part. The transmitted power  $P_t$  is defined as the power consumed in the transmitter nanocoil and the received power  $P_r$  is defined as the power consumed in the load impedance  $Z_L$ . To maximize the received power  $P_r$ , the load impedance is adjusted to be equal with the complex conjugate of the total impedance of the receiver nanocoil, i.e.,  $Z_L = Z_2^* + Z_{12}$ . Hence, the received power becomes half of the total power consumed in the receiver nanocoil, i.e.,  $P_r = \text{Re}\{|V_M|^2/4(Z_2 + Z_{12} + Z_L)\}$ . After some algebraic manipulations, the path loss in the NMI communication channel is found as

$$\frac{P_t}{P_r} = 2 + 4 \frac{R_1 R_2}{\omega^2 M^2}. \quad (4)$$

We assume that the transmitter and receiver nanocoils are identical. Thus,  $R_1 = R_2 = R$ ,  $C_1 = C_2 = C$ , and  $L_1 = L_2 = L$ . In [13], the self-inductance of a square planar nanocoil having a magnetic core is given by

$$L = 1.17\mu \frac{N^2(a^o + a^i)}{1 + 2.75 \left( \frac{a^o - a^i}{a^o + a^i} \right)} \quad (5)$$

where  $a^o$  and  $a^i$  are the lengths of the inner and outer sides of the square planar nanocoil, respectively, as illustrated in Fig. 1(b),  $N$  is the number of turns of the nanocoil, and  $\mu$  is the magnetic permeability of the magnetic core of the nanocoil. The mutual inductance  $M$  between the transmitter and receiver nanocoils is expressed as

$$M = k\sqrt{L_1 L_2} = kL \quad (6)$$

where  $k$  is the coupling coefficient between the nanocoils. The coupling coefficient can be approximated as [9]

$$k = \frac{a_{avg}^3}{8 \left( \sqrt{d^2 + a_{avg}^2/4} \right)^3} \cos(\alpha) \quad (7)$$

where  $d$  is the communication distance, i.e., the distance between transmitter and receiver nanocoils,  $\alpha$  is the angle between the axes of the coupled coils, and  $a_{avg}$  is the average conductor side length of the square planar nanocoils, expressed as  $a_{avg} = (a^i + a^o)/2$ . The resistance  $R$  of the nanocoils is found as follows

$$R = 4a_{avg}R_0N \quad (8)$$

where  $R_0$  is the resistance of the unit length of the conductor wire and it is given by  $R_0 = \rho/(wt)$  where  $\rho$  is the resistivity of the conductor wire,  $w$  and  $t$  are the width and thickness of the conductor wire, respectively, as shown in Fig 1(b). Assuming  $d \gg a_{avg}$ , the path loss expression becomes

$$\frac{P_t}{P_r} \approx \frac{748R_0^2(1 + 2.75\Gamma)^2 d^6}{\omega^2 a_{avg}^6 \mu^2 N^2 \cos^2(\alpha)} \quad (9)$$

where  $\Gamma = (a^o - a^i)/(a^o + a^i)$ . The path loss is  $6^{th}$ -order function of  $d$ . Thus, an increase in the communication distance greatly reduces the received power. On the other hand, the received power can be increased by using a large signal frequency  $\omega$ , a large number of turns  $N$ , a large average

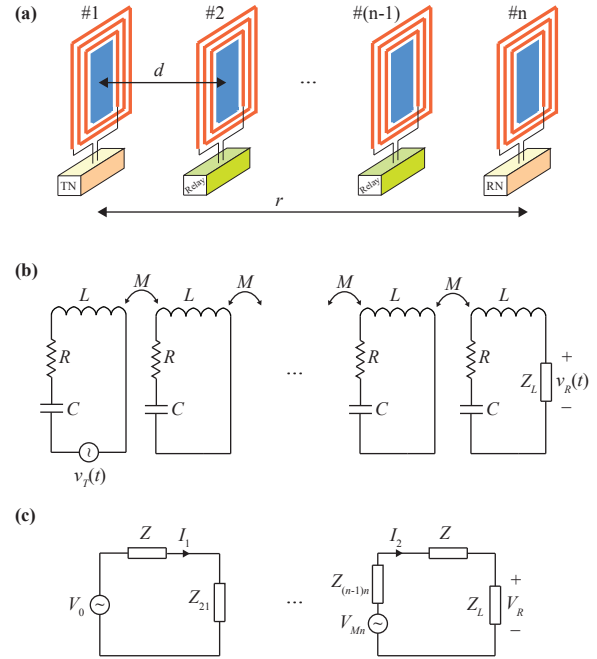


Fig. 2. The waveguide model for the NMI communication, and circuit models of the NMI communication. (a) The model of the NMI waveguide communication. (b) The transformer model. (c) The equivalent circuit of the transformer model.

nanocoil side length  $a_{avg}$ , a large permeability of the magnetic core  $\mu$ , and a small unit length resistance  $R_0$ .

### III. WAVEGUIDE MODEL FOR NMI COMMUNICATION

In the previous section, the NMI communication channel was modeled for a point-to-point communication network. In the NMI communication, the received power falls off proportionally with  $d^{-6}$  in (9). Therefore, the communication distance  $d$  has a severe effect on the received power and this effect limits the NMI communication range. In this section, to increase the range of the NMI communication, we employ relay nanonodes between the transmitter and receiver nanocoils to form a magneto-inductive waveguide.

For the NMI communication, the relay nanonodes are assumed to be passive devices; that is, a relay nanonode includes only a nanocoil and does not have a power source or processing circuitry. The signal propagation through the relay nanonodes is achieved by the magnetic coupling between nanocoils. That is, the sinusoidal current passing in the transmitter nanocoil induces a sinusoidal current in the nanocoil of the first relay nanonode. Then, the induced sinusoidal current in the first relay nanocoil also induces a sinusoidal current in the second relay nanocoil and the transmitted signal propagates in a similar manner until the induced current reaches the receiver nanodevice.

The waveguide model for the NMI communication is shown in Fig. 2(a). In the waveguide, there are  $n$  nanocoils equally spaced including the nanocoils in the transmitter and receiver nanodevices. Therefore, if the distance between transmitter and

receiver nanocoils is  $r$ , the distance between two successive nanocoil is given by  $d = r/(n-1)$ . Furthermore, we assume that only the adjacent nanocoils are magnetically coupled; hence, we only use the mutual inductance between the adjacent nanocoils.

The multi-stage transformer model for the NMI communication waveguide is demonstrated in Fig 2(b). The nanocoils in all nodes are assumed to be identical, and  $L$ ,  $M$ , and  $R$  are given in (5), (6), and (8), respectively. The equivalent circuit for the transmitter and receiver nanocoils can be seen in Fig 2(c) where

$$\begin{aligned} Z &= R + j\omega L + 1/j\omega C \\ Z_L &= Z^* + Z_{(n-1)n}^* \\ Z_{i(i-1)} &= \frac{\omega^2 M^2}{Z + Z_{(i+1)i}}; \quad i = 2, \dots, n \text{ and } Z_{(n+1)n} = Z_L \\ Z_{(i-1)i} &= \frac{\omega^2 M^2}{Z + Z_{(i-2)(i-1)}}; \quad i = 3, \dots, n \text{ and } Z_{12} = \frac{\omega^2 M^2}{Z} \\ V_{Mi} &= -j\omega M \frac{V_{M(i-1)}}{Z + Z_{(i-2)(i-1)}}; \quad i = 2, \dots, n \text{ and } V_{M1} = V_0 \end{aligned} \quad (10)$$

where  $Z_{ij}$  denotes the reflected impedance of the  $i^{\text{th}}$  nanocoil into the  $j^{\text{th}}$  nanocoil, and  $V_{Mi}$  denotes the induced voltage on the  $i^{\text{th}}$  nanocoil. According to the equivalent circuit of the NMI communication waveguide, the transmitted power  $P_t$  and the received power  $P_r$  are expressed as

$$\begin{aligned} P_t &= \frac{1}{2} \text{Re} \left\{ \frac{|V_0|^2}{Z + Z_{21}} \right\} \\ P_r &= \frac{1}{2} \text{Re} \left\{ \frac{|V_{Mn}|^2}{2(Z_L + Z + Z_{(n-1)n})} \right\}. \end{aligned} \quad (11)$$

To maximize the received power, the angle frequency of the transmitted signal is chosen the same as the resonant frequency of the equivalent RLC circuit of the planar nanocoil. The resonant frequency of the nanocoil is given as  $\omega_0 = 1/\sqrt{LC}$  and hence  $j\omega_0 L + (1/j\omega_0 C) = 0$ . Therefore, the impedance of a nanocoil becomes  $Z = R$  and the path loss in the NMI communication waveguide is given by

$$\frac{P_t}{P_r} = \frac{2V_0^2}{|V_{Mn}|^2} \left( \frac{R + Z_{(n-1)n} + Z_L}{R + Z_{21}} \right) \quad (12)$$

where

$$\begin{aligned} Z_{(n-1)n} &= \frac{\omega_0^2 M^2}{R + \frac{\omega_0^2 M^2}{R + \frac{\omega_0^2 M^2}{\ddots + Z_{12}}}} \\ Z_{21} &= \frac{\omega_0^2 M^2}{R + \frac{\omega_0^2 M^2}{R + \frac{\omega_0^2 M^2}{\ddots + Z_L}}} \end{aligned} \quad (13)$$

and  $Z_{12} = \omega^2 M^2/R$ , and  $Z_L = R + Z_{(n-1)n}$ . Since the expression in (12) is too complicated to simplify, the effect of the communication distance  $r$  on the path loss in the waveguide is analyzed numerically in the next section.

#### IV. PERFORMANCE EVALUATION

In this section, we present the numerical performance analysis of both the point-to-point NMI communication and the NMI communication waveguide. The path loss, i.e.,  $(P_t/P_r)$ , is used as the performance criterion and evaluated with respect to the communication distance for different magnetic permeabilities of the magnetic cores in the nanocoils. We use MATLAB to perform the performance analysis. The path loss expressions for the point-to-point NMI communication and the NMI communication waveguide are given in (9) and (12), respectively. For the dimensions of a nanocoil, we use  $a^o = 500\text{nm}$ ,  $w = 40\text{nm}$ ,  $s = 1\text{nm}$ ,  $t = 40\text{nm}$ , with  $N = 5$  turns and  $\alpha = 0^\circ$ . For the numerical analysis, we consider the conductor wire as copper whose resistivity is  $\rho = 1.68 \times 10^{-8} \Omega\text{m}$ . Furthermore, the angle frequency of the signal source is assumed to be  $\omega = 1/\sqrt{LC}$  where  $L$  is given in (5) and the capacitance is  $C = 0.01\text{fF}$ .

For the point-to-point NMI communication, the path loss in dB with respect to the communication distance  $d$  for different relative magnetic permeabilities  $\mu_r$  is illustrated in Fig. 3. The relative magnetic permeability is defined as  $\mu_r = \mu/\mu_0$  where  $\mu_0$  is the magnetic permeability of free space. The path loss increases with an increase in the communication distance as seen in (9) and in Fig. 3. For example, for  $\mu_r = 1000$ , the path loss is 63.4dB at  $d = 10\mu\text{m}$  and the path loss is 123.0dB at  $d = 100\mu\text{m}$ . As the communication distance increases, first, the path loss exhibits a fast increase and then, it slowly increases. As a result, considering the power limitation of the nanodevices, the practical communication range of the point-to-point NMI communication is short.

According to the results shown in Fig. 3, a decrease in the relative permeability  $\mu_r$  increases the path loss, which also can be seen in (9). For  $d \geq 5\mu\text{m}$ , decreasing  $\mu_r$  from 1000 to 300 increases the path loss by 5dB. Therefore, using a magnetic core with large permeability in the nanocoil improves the achievable communication range of the NMI communication by decreasing the path loss.

For the NMI waveguide communication, the path loss in dB with respect to the communication distance between transmitter and receiver  $r$  for different relative magnetic permeabilities  $\mu_r$  is demonstrated in Fig. 4. In this analysis, the distance between adjacent coils is constant and given by  $d = a^o/2$ . The communication distance is increased by increasing the number of relay coils in the waveguide. In addition, the relay coils in the waveguide do not require any power. The results show that an increase in the communication distance increases the path loss. For instance, for  $\mu_r = 1000$ , the path loss is 3.3dB at  $r = 10\mu\text{m}$  and the path loss is 19.0dB at  $r = 100\mu\text{m}$ . Note that, for the same communication distances, the waveguide technique greatly reduces the path loss compared with the point-to-point NMI communication.

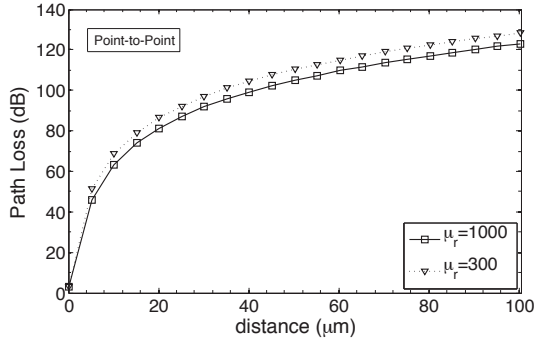


Fig. 3. The path loss for the point-to-point NMI communication with respect to the communication distance  $d$  for different relative permeability values.

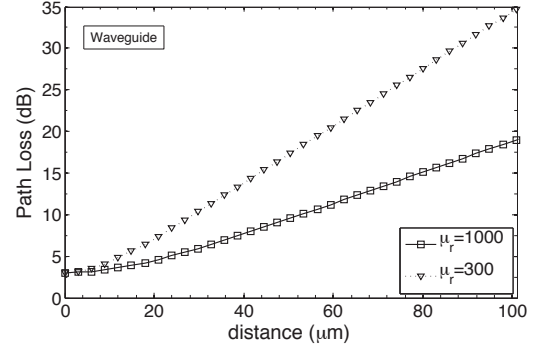


Fig. 4. The path loss for the NMI waveguide communication with respect to the communication distance  $r$  for different relative permeability values.

Although the path loss in dB abruptly increases in the point-to-point NMI communication as the communication distance increases, in the NMI waveguide, the path loss in dB increases linearly with a small slope compared with the point-to-point case. Thus, by using the waveguide technique, the range of the NMI communication is significantly increased. Based on the results, the NMI waveguide communication stands as a promising solution to long range nanoscale communication.

The numerical results given in Fig. 4 show that, for the NMI waveguide communication, a decrease in the relative permeability  $\mu_r$  increases the path loss. As the communication distance increases, the difference between the path loss values for  $\mu_r = 300$  and  $\mu_r = 1000$  increases. Even though for  $d = 20\mu\text{m}$ , decreasing  $\mu_r$  from 1000 to 300 increases the path loss by 2.8dB, for  $d = 100\mu\text{m}$ , decreasing  $\mu_r$  from 1000 to 300 increases the path loss by 15.7dB. Hence, a large permeability of the core in the nanocoil significantly improves the feasible communication range of the NMI communication by decreasing the path loss.

## V. CONCLUSION

In this paper, we propose a novel nanoscale communication technique, i.e., the NMI communication which relies on the magnetic coupling between nanocoils. We present a realistic physical communication model for both the point-to-point and waveguide NMI communication methods. Then, we derive the closed-form expression of the path loss for both techniques. The numerical analyses show that using waveguide method in the NMI communication can significantly reduce the path loss and increase the achievable communication range of the NMI communication. Since the relay coils used in the waveguide do not require power, a single passive nanocoil can serve as a relay nanonode. In addition, using a magnetic core in the nanocoils with large permeability also decreases the path loss in the NMI communication.

The problems that EM waves encounter at nanoscale are high attenuation rates due to high molecular absorption and frequency selective channel characteristics. The novel NMI communication overcomes these problems by introducing low absorption losses and flat channel characteristics. Although the path loss is more severe in the NMI communication than

in nanoscale EM communication, using the waveguide NMI communication method solves the high path loss problem in the NMI communication. Therefore, the NMI communication stands as a promising alternative wireless nanoscale communication technique.

## REFERENCES

- [1] S. Hiyama, Y. Moritani, T. Suda, R. Egashira, A. Enomoto, M. Moore and T. Nakano, "Molecular Communication," in *Proc. of NSTI Nanotech. 2005*, Anaheim, California, USA, 2005.
- [2] M. Gregori, I. F. Akyildiz, "A new nanonetwork architecture using flagellated bacteria and catalytic nanomotors," *IEEE JSAC*, vol. 28, no. 4, pp. 612-619, 2010.
- [3] I. F. Akyildiz, F. Brunetti, C. Blazquez, "Nanonetworks: A new communication paradigm," *Computer Networks (Elsevier)*, vol. 52, no. 12, pp. 2260-2279, 2008.
- [4] R. A. Freitas. "Nanomedicine, Vol. I: Basic Capabilities," Landes Bioscience, 1999.
- [5] D. Kilinc, O. B. Akan, "An Information Theoretical Analysis of Nanoscale Molecular Gap Junction Communication Channel Between Cardiomyocytes," *IEEE Transactions on Nanotechnology*, vol.12, no.2, pp.129-136, March 2013.
- [6] D. Kilinc, O. B. Akan, "Receiver Design for Molecular Communication," to appear in *IEEE Journal on Selected Areas in Communications (JSAC)*, 2013.
- [7] I. F. Akyildiz, J. M. Jornet, and M. Pierobon, "Propagation Models for Nanocommunication Networks," in *Proc. of EUCAP 2010, Fourth European Conference on Antennas and Propagation*, April 2010, pp. 4229-4232.
- [8] R. M. Goody and Y. L. Yung, *Atmospheric Radiation: Theoretical basis*, 2nd ed. Oxford University Press, 1989.
- [9] K. Finkenzeller, *RFID Handbook: Radio-Frequency Identification Fundamentals and Applications*, 2nd ed: Wiley, 2004.
- [10] Z. Sun and I. Akyildiz, "Magnetic induction communications for wireless underground sensor networks," *IEEE Transactions on Antenna and Propagation*, vol. 58, no. 7, pp. 2426-2435, July 2010.
- [11] B. Gulbahar, O. B. Akan, "A Communication Theoretical Modeling and Analysis of Underwater Magneto-Inductive Wireless Channels," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3326-2234, 2012.
- [12] M. Sun, S. A. Hackworth, Z. Tang, G. Gilbert, S. Cardin, and R. J. Sclabassi, How to pass information and deliver energy to a network of implantable devices within the human body, in *Proc. IEEE Conf. on Engineering in Medicine and Biology Society (EMBS 2007)*, Aug. 2007, pp. 5286-5289.
- [13] S. S. Mohan, M. del Mar Hershenson, S. P. Boyd, T. H. Lee, "Simple accurate expressions for planar spiral inductors," *IEEE J. Solid-State Circuits*, vol. 34, pp. 1419-1424, Oct. 1999.