

Synaptic Interference Channel

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Abstract—Synaptic channels automatically adapt their weights to compensate for the variations resulted from the input and output characteristics, i.e., spike frequency, time correlation among inputs, time difference between presynaptic and postsynaptic action potentials. Modification of the synaptic conductances, i.e., channel weights, is the main mechanism that enables learning in neurons. In this paper, we approach this learning mechanism from a different perspective. First, we analyze the single-input single-output (SISO) and multi-input single-output (MISO) synaptic interference channels, and achievable communication rates. Furthermore, we provide the natural adaptive weight update algorithm for neurons based on experimental findings. Our results demonstrate that neurons are capable of mitigating the interference, and achieve rates close to the capacity.

I. INTRODUCTION

Neuro-spike communication, i.e., the communication of spiking neurons through the release of neurotransmitter molecules to the synaptic clefts, is an interdisciplinary research area, which combines the fields of neuroscience, communications and nanotechnology [1]. The point-to-point synaptic communication channel is studied in the literature widely as in [1] and [2], and the multiple-access neuro-spike communication channel is studied in [3]. Development of these channel models is promising for future nanoscale and molecular communication techniques and applications.

Some current work concentrates on scheduling on neurons, to minimize the interference in the neuron topology [4], and to optimize the signaling schedule for nanomachines in in-body sensor-actuator networks [5]. However, considering the essential role of synaptic connections in memory and learning, more analysis is needed on neuro-spike communication.

Synapses are able to adjust their channel conductances depending on the action potential (AP) characteristics, called as neural plasticity (NP) in the neuroscience literature [6]. NP aims at building stronger connections among neurons so that actively transmitting neurons are sustained to carry information, but on the other hand, other connections, which are usually not correlated with most of the transmitted information, fade away. In this paper, we investigate the sustainability of the synaptic connections from a communication theory perspective. We treat the synapses that do not provide

useful information transmission as interfering connections. Information transfer via interfering connections should be cancelled out at the output neuron. To achieve this task, neurons, through feedback mechanisms, manage to adjust the synaptic conductances, i.e., synaptic weights. This mechanism among neuronal connections enables the reduction of interference caused by uncorrelated synapses. In this study, we investigate the optimality of this adaptive interference canceling method. To the best of our knowledge, the effect of interference among a cluster of neurons has not been investigated yet.

In this paper, we consider the most basic model of a neuron, which consists of an input with some synaptic weight vector and an activation function or transfer function inside the neuron determining output. We model neurons as linear systems, and then, analyze their transmission performance. Here, we compare the Signal-to-Noise plus Interference Ratio (SNIR) obtained for actual interference cancellation mechanism seen in neurons and to the maximum achievable SNIR. The results show that neurons are actually good interference canceling elements that are already available in the nature. This finding could be exploited to build strong connections enabling the transfer of most common data seen in the input side.

The remainder of this paper is organized as follows. In Section II, we give a background on neural signaling and communication. In Section III, we provide the discrete linear system models for the SISO and MISO synaptic interference channels by projecting the stochastic processes, i.e., inputs and the channel noise, onto interference eigenfunctions. Then, we investigate and analyze the SISO and MISO synaptic interference channels. In Section IV, we provide the maximum information rate for single-input and multiple-access neuron interference channels. In Section V, by explaining the neural learning algorithm through synaptic weight modification, i.e., STDP, we interpret the output neuron SIR, analytically. In Section VI, we provide the performance results of optimal detection and STDP algorithms on the output SIR, and interpret the results. Finally, we conclude the paper in Section VII.

II. BACKGROUND AND SYSTEM MODEL

In this section, we provide the essential components of neural communication. These include the *transmitting node*, *transmission medium*, and the *receiving node*.

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A. Transmitting Node: The Presynaptic Neuron

When a random stimulus is applied, the presynaptic neuron terminal generates a Poisson distributed AP train, $S(t)$, at its axon, which has a firing rate λ . Neuronal response characteristics are usually characterized by Linear-Nonlinear-Poisson (LNP) model. The details of the firing and AP train generation can be found in [3]. AP train, i.e., $S(t)$, triggers the *vesicle release process* from the presynaptic terminal. The release process is shaped according to the rate of AP generation, λ .

B. Transmission Process: The Synaptic Channel

AP generated by the presynaptic terminal enables vesicle release to the synaptic channel. Release amount depends on the strength of the AP train, the size of available vesicle pool located in the neural terminal, and the quantal release parameters. These concepts are explained in [8].

Input the synaptic channel, $S(t)$, is scaled by the synaptic conductance. Synaptic channels have variable conductances. These conductances, i.e., weights, are adjusted through spike-timing-dependent plasticity (STDP), which will be discussed in Section V. In this paper, the synaptic weight is denoted by w , which is to be used in our linear system model.

C. Receiving Node: The Postsynaptic Neuron

At the postsynaptic terminal, each input propagated through the soma is summed linearly in the point neuron, i.e., perceptron, model. Perceptron can be described by

$$r_i(t) = h_i S_i(t) + Z_i(t), \quad h_i = \sqrt{P_i} \beta_i w_i, \quad (1)$$

where $Z_i(t)$ is assumed to be an independent interference stochastic waveform that may be composed of both thermal noise and interfering signals of other transmitting neurons, i.e., presynaptic terminals. When $Z_i(t)$ is composed of known waveforms in addition to independent Gaussian noise

$$Z_i(t) = \sum_{j \neq i}^M \sqrt{P_j} \beta_j \alpha_{ij} S_j(t) + N(t). \quad (2)$$

In (2), $Z_i(t)$ is the interference signal seen at the soma [9], α_{ij} 's and $S_j(t)$'s are the synaptic interference weights and unit power synaptic inputs, respectively. β_j denotes whether the inputs coming from either EXS or INS. It is bipolar, 1 if the synapse is an EXS, -1 if an INS. P_j is the input power at the j^{th} synaptic connection, and finally $N(t)$ stands for the axonal noise, and $M = M_{EX} + M_{IN}$, where M_{EX} and M_{IN} are for the total number of EXSs and INSs, is for the total number of EXSs and INSs to the soma.

From the perspective of the neuron i receiving the input signal $S_i(t)$, the interference $Z_i(t)$ is simply a stochastic process, which we assume zero mean without loss of generality.

III. SYNAPTIC INTERFERENCE CHANNEL

In this section, based on the *interference eigenfunctions* concept, we present a discrete-time linear neuron model. Then, using this linear model, we first investigate the SISO neuron interference channel, and then, we analyze the MISO neuron interference channel.

A. SISO Neuron Synaptic Interference Channel

In the single-input neuron channel, the receiver neuron i , i.e., the i^{th} postsynaptic terminal axon, observes the signal $r_i(t) = h_i S_i(t) + Z_i(t)$ on the interval $[0, T]$. Projecting the received signal onto the interference eigenfunctions $\Phi_1(t), \dots, \Phi_N(t)$ as described in [7], we obtain the vector output

$$\tilde{\mathbf{r}}_i = h_i \tilde{\mathbf{s}}_i + \tilde{\mathbf{z}}_i, \quad (3)$$

where $\tilde{\mathbf{s}}_i$ and $\tilde{\mathbf{z}}_i$ have n^{th} components $s_{in} = \langle S_i(t), \Phi_n(t) \rangle$, $z_{in} = \langle Z_i(t), \Phi_n(t) \rangle$ and the z_{in} 's are mutually uncorrelated. In (3), $\tilde{\mathbf{s}}_i$ is an $N \times 1$ vector with user's signature, and h_i is the channel gain parameter equal to $h_i = \sqrt{P_i} \beta_i w_{ii}$, and $\tilde{\mathbf{z}}_i = [z_1 \ z_2 \ \dots \ z_N]^T$ is the channel interference vector combined with the zero mean AWGN vector with covariance matrix $\sigma^2 \mathbb{I}_N$ with size $N \times N$, and $\sigma^2 = E[n_k^2] = N_0/2$.

Here, to simplify the analysis, we first work with unit vectors, and then, generalize our analysis to non-unit vectors. With no loss of generality we assume that the basis functions $\Phi_n(t)$ also span the signal space for $S_i(t)$, and $h_i \tilde{\mathbf{s}}_i$ contains all available information about $h_i S_i(t)$.

B. MISO Neuron Synaptic Interference Channel

When there are multiple presynaptic neuron terminals sending APs simultaneously, the receiver neuron i , i.e., the postsynaptic terminal i^{th} axon, observes a summation of signals

$$r_i(t) = \sum_{j=1}^M h_j S_j(t) + N(t) \quad (4)$$

as input on the interval $[0, T]$, where $h_j = \sqrt{P_j} \beta_j w_{ij}$. For multiple presynaptic terminal case, projecting the received signal onto the interference eigenfunctions $\Phi_1(t), \dots, \Phi_N(t)$, we obtain the following linear relation for the vector output

$$\tilde{\mathbf{r}}_i = \mathbf{S} \tilde{\mathbf{h}}_i + \tilde{\mathbf{n}}_i, \quad (5)$$

where $\mathbf{S} = [\tilde{\mathbf{s}}_1 \ \tilde{\mathbf{s}}_2 \ \dots \ \tilde{\mathbf{s}}_M]$ is an $N \times M$ matrix with users' signatures, and $\tilde{\mathbf{s}}_j$ is the projection component of $S_j(t)$ onto interference eigenfunctions, and $\tilde{\mathbf{h}}_i = [h_{i1} \ h_{i2} \ \dots \ h_{iM}]^T$, is the channel gain vector where $h_{ij} = \sqrt{P_j} \beta_j w_{ij}$, and $\tilde{\mathbf{z}}_i = [z_1 \ z_2 \ \dots \ z_N]^T$ is the channel interference vector combined with the AWGN vector with zero mean and covariance matrix $\sigma^2 \mathbb{I}_N$ of size $N \times N$, and $\sigma^2 = E[n_k^2] = N_0/2$.

IV. ACHIEVABLE COMMUNICATION RATES IN SYNAPTIC INTERFERENCE CHANNEL

As neurons have very large-scale connections, and multiple neurons simultaneously transfer data through synaptic connections, they suffer from interference. Herein, we investigate the capacity of single-input and multiple-access synaptic interference channels using the linear system model developed in Section III and provide the output SNIR for both cases.

A. SISO Neuron Synaptic Interference Channel

1) *Total Signal Power at the Output:* Total signal power at the output node can be found using \mathbf{r}_i as $\|\mathbf{r}_i\|_2^2 = \mathbf{r}_i^\top \mathbf{r}_i = h_i^2 \sum_{k=1}^N s_{ik}^2 + 2h_i \sum_{k=1}^N s_{ik} z_{ik} + \sum_{k=1}^N z_{ik}^2$. Hence, the SNIR at the output axon hillock i is computed as

$$\gamma_{out,i} = \frac{\mathbb{E} \left[h_i^2 \sum_{k=1}^N s_{ik}^2 \right]}{\mathbb{E} \left[2h_i \sum_{k=1}^N s_{ik} z_{ik} + \sum_{k=1}^N z_{ik}^2 \right]} = \frac{w_{ii}^2 P_i}{\mathbb{E} \left[\text{Tr}(\mathbf{z}\mathbf{z}^\top) \right]} \sum_{k=1}^N s_{ik}^2, \quad (6)$$

where we assume that interference signal is independent from the input signal, and zero mean, and (6) depends on the channel weight, i.e., w_{ii} , which changes adaptively depending on the AP patterns. Hence, the time dependent output SIR can be calculated using the algorithm described in Section V.

2) *Channel Capacity:* Using (6), for the node at the output side, and assuming unit power constraint on the presynaptic terminal, we obtain the following rate expression

$$R_i = \frac{1}{2} \log(1 + \gamma_{out,i}) = \frac{1}{2} \log \left(1 + \frac{w_{ii}^2 P_i}{\sum_{j \neq i}^M P_j \alpha_{ij}^2 + \sigma^2 N} \right). \quad (7)$$

Using (3), and assuming no synaptic interference, the capacity expression for the single-input neural communication channel can be simplified to AWGN channel with the capacity

$$C(\underline{s}; \underline{r}) = \max_{p(\underline{s})} I(\underline{s}; \underline{r}) = \frac{1}{2} \log \left(1 + \frac{w_{ii}^2 P_i}{\sigma^2 N} \right). \quad (8)$$

B. MISO Neuron Synaptic Interference Channel

1) *Total Signal Power at the Output:* Total signal power at the output node can be found using \mathbf{r}_i as $\|\mathbf{r}_i\|_2^2 = \mathbf{h}_i^\top \mathbf{S}^\top \mathbf{S} \mathbf{h}_i + 2\mathbf{h}_i^\top \mathbf{n}_i + \mathbf{n}_i^\top \mathbf{n}_i$. Thus, the SNIR at the output axon hillock is

$$\begin{aligned} \gamma_{out,i} &= \frac{\mathbb{E} \left[\mathbf{h}_i^\top \mathbf{S}^\top \mathbf{S} \mathbf{h}_i \right]}{\mathbb{E} \left[2\mathbf{h}_i^\top \mathbf{n}_i + \mathbf{n}_i^\top \mathbf{n}_i \right]} = \frac{\text{Tr}(\mathbf{S} \mathbf{E}[\mathbf{h}_i \mathbf{h}_i^\top] \mathbf{S}^\top)}{\text{Tr}(\mathbf{E}[\mathbf{n}_i \mathbf{n}_i^\top])} \\ &= \frac{1}{\text{Tr}(\mathbf{E}[\mathbf{n}_i \mathbf{n}_i^\top])} \sum_{k=1}^N w_{ik}^2 P_k \text{Tr}(\mathbf{s}_k \mathbf{s}_k^\top). \end{aligned} \quad (9)$$

(9) depends on the channel weights, i.e., w_{ik} 's, which change adaptively depending on the AP patterns. Using the algorithm for modification of the synaptic weight as described in Section V, the time dependent SIR at the output neuron could be iteratively calculated.

2) *Channel Capacity:* Using (9), for the node at the output side, and assuming unit power constraint on the presynaptic terminals, we obtain the following rate expression

$$R_i = \frac{1}{2} \log(1 + \gamma_{out,i}) = \frac{1}{2} \log \left(1 + \frac{\sum_{k=1}^M w_{ik}^2 P_k}{\sigma^2 N} \right). \quad (10)$$

Using the expression in (5), and assuming no synaptic interference, the capacity expression for the multiaccess neural

TABLE I
FUNCTION AND VARIABLE DEFINITIONS.

Definition	Symbol	Value / Range
EXS, INS conductances	$g_{ex}(t), g_{in}(t)$	—
EXS, INS time constants	τ_{EX}, τ_{IN}	5 ms, 5 ms
AWGN at axon, mean and std	μ_N, σ_N	0, $\mathcal{O}(10^{-4})$
Peak magnitude of the EPSP waveform	h_p	50 mV
Time at which EPSP reaches its peak	t_p	0.5 ms
Synaptic strength decrease function	$M(t)$	—
Synaptic strength increase function	$P_a(t)$	—
Interspike interval strengthening range	τ_+	20 ms
Interspike interval weakening range	τ_-	20 ms
Max amount of synaptic modification	A_-	1.05A+
Max amount of synaptic modification	A_+	0.005
% of synaptic conductance modification	$g(\delta t)$	[-0.5, 0.5]
Peak synaptic conductance	\bar{g}_{\max}	0.035
Peak EXS conductance	\bar{g}_a	$0 \leq \bar{g}_a \leq \bar{g}_{\max}$
Peak INS conductance	\bar{g}_{in}	$0 \leq \bar{g}_{in} \leq \bar{g}_{\max}$

communication channel can be simplified to AWGN channel with the capacity

$$C(\mathbf{S}; \underline{r}) = \max_{p(\mathbf{S})} I(\mathbf{S}; \underline{r}) = \frac{1}{2} \log \det \left(\frac{\mathbf{S} \mathbf{P} \mathbf{S}^\top + \mathbb{I}_N \sigma^2}{\sigma^2} \right), \quad (11)$$

where \mathbf{P} is the power matrix obtained as $\mathbf{P} = \mathbb{E}[\mathbf{h}_i \mathbf{h}_i^\top] = \text{diag}(w_{i1}^2 P_1, \dots, w_{iM}^2 P_M)$, because we assume $\beta_j = 1$ or $\beta_j = -1$ with equal probability, and each synapse could be either excitatory or inhibitory independent of each other. Hence, $\mathbb{E}[\beta_i] = 0$, and $\mathbb{E}[\beta_i \beta_j] = \delta_{ij}$.

V. SYNAPTIC CONDUCTANCE MODIFICATION

In this section, we focus on the actual characteristics of synaptic weights. First, we introduce the mechanism behind the synaptic weight modification. Then, we incorporate the factors, such as synaptic strength decrease and increase functions, synaptic conductance modification characteristics, that enable us to understand these changes, and build an algorithm to describe the synaptic weight update mechanism. Later, in the performance evaluation, in Section VI, we compare the ideal synaptic communication rate to actual adaptive communication between neuron terminals. Hence, this section provides a mean to understand the tradeoffs between optimum and actual achievable transmission performance among multiple-access synaptic connections.

Spike-timing-dependent plasticity (STDP) is a biological process that adjusts the strength of connections between neurons in the brain. The process adjusts the connection strengths based on the relative timing of a particular neuron's output and input APs (or spikes) [6].

If an input spike to a neuron occurs immediately before that neuron's output spike, then that particular input is made stronger. If an input spike occurs immediately after an output spike, then that particular input is made weaker as a result of the STDP process. The process continues until a subset of the initial set of connections remain, while the influence of all others is reduced to 0.

Let Δt be a random variable standing for the time difference between presynaptic potential and postsynaptic potential at a neuron. We know the relation between $\Delta t = \delta t$ and synaptic

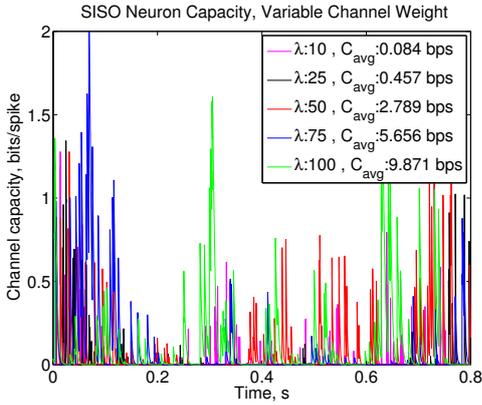


Fig. 1. SISO neuron channel capacity varying with time, $\sigma^2 = 2.5 \times 10^{-8}$.

modification. $\bar{g}_{\max}g(\delta t)$ is the modification amount in the peak synaptic conductance. $g(\delta t)$ determines the amount of synaptic modification arising from a single pair of pre- and postsynaptic spikes separated by a time δt . Hence, at each time step δt , the peak synaptic conductance is modified as $\bar{g}_{\max}(t + \delta t) = \bar{g}_{\max}(t) + \bar{g}_{\max}g(\delta t)$, where $g(\delta t)$ is the percentage of modification.

In Algorithm 1, we summarize the synaptic weight modification characteristics. The definitions of the functions and variables in Algorithm 1 are given in Table I. The details of the synaptic modification are described in [11].

VI. PERFORMANCE EVALUATION

In this section, we analyze the SISO and MISO synaptic channels under interference, respectively. First, we evaluate the estimation performance of the SISO and MISO synaptic interference channels and evaluate the MSE for the channel weight estimations. Furthermore, we investigate the SNIR performance of both channels and their achievable rates.

A. SISO Synaptic Interference Channel Performance Analysis

The neuron synaptic weights are automatically updated by the synaptic channel itself according to spike timing characteristics. This is also summarized in Algorithm 1 in Section V. Using the algorithm, we analyze how the capacity for the SISO interference channel changes adaptively for a low axonal noise variance of $\sigma^2 = 2.5 \times 10^{-8}$. In Fig. 1, we illustrate the time course of the capacity in bits/spike for the SISO channel for different AP generation rates. As seen from the figure, the average capacity of the channel is proportional to the AP generation rate. Higher rates are achievable as long as the presynaptic neuron terminal is not saturated.

We analyze the SNIR and the channel capacity for the SISO synaptic interference channel. The performance result for the communication rate is shown in Fig. 2 for varying ratios of M_{EX} to M_{IN} . As the M_{EX}/M_{IN} ratio increases, the interference on the SISO synaptic interference channel increases. Hence, the communication rate drops. For $\sigma^2 = 2.5 \times 10^{-8}$, for small number of interfering presynaptic inputs, communication rates around 3 bits/channel use are achievable.

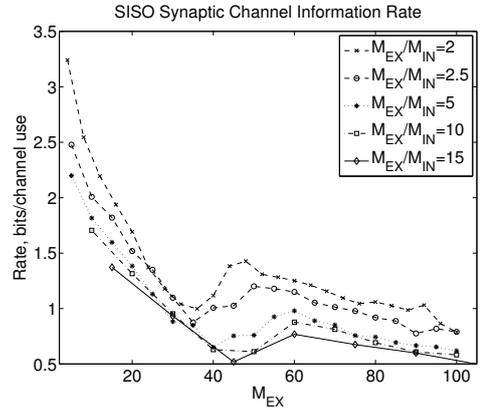


Fig. 2. Communication rate of the SISO neuron interference channel for various M_{EX} and M_{IN} values, $\sigma^2 = 2.5 \times 10^{-8}$.

B. MISO Synaptic Interference Channel Performance Analysis

In this section, extending the interference analysis for SISO synaptic interference channel to MISO synaptic interference channel, we analyze the interference depending on the synaptic weights, i.e., strengths. We assume that at least one presynaptic input is strong, i.e., at least one input has $w = \bar{g}_{\max}$. In the performance analysis, other inputs are given weights $w = \alpha\bar{g}_{\max}$, where $\alpha < 1$. The number of strong and weak presynaptic inputs are denoted by N_S and N_W , respectively.

In Fig. 3, we illustrate how the communication rate for the MISO channel changes depending on the variable synaptic strengths of multiple users and the number of M_{EX} and axonal noise variance. In Fig. 3(a), the communication rate in bits/channel use for $\sigma = 10^{-4}$ is investigated for varying number of presynaptic inputs. The presynaptic inputs are assigned an average weight of $w = \alpha\bar{g}_{\max}$, where $\alpha = [0.1, 0.7]$ is incremented with a step size 0.2. As α increases, the total weight of the presynaptic inputs, and the output rate increases. The analysis is repeated for $\sigma = 10^{-3}$, $\sigma = 10^{-2}$ in Fig. 3(b) and 3(c), respectively. Performance analysis points out the downward trend of the total rate of synaptic communication as the axonal variance increases.

In Fig. 4, using Algorithm 1 presented in Section V, the time course of MISO synaptic channel capacity is investigated. Using the algorithm, we analyze how the capacity for the MISO interference channel changes adaptively for a low axonal noise variance of $\sigma^2 = 2.5 \times 10^{-8}$. In Fig. 4, we illustrate the time course of the capacity in bits/spike for the MISO channel for AP generation rate $\lambda = 10$ spike/s for different M_{EX} values. As seen from the figure, the average capacity of the communication channel is incremented as M_{EX} increases, and higher rates are achievable at the postsynaptic neuron terminal. Comparing Fig. 4 to Fig. 3, as the number of excitatory synapses increases, MISO synaptic channel performs close to the maximum channel capacity indicated in Fig. 3(a).

VII. CONCLUSION

In this paper, we investigate the effect of interference among the presynaptic terminals in SISO and MISO synaptic

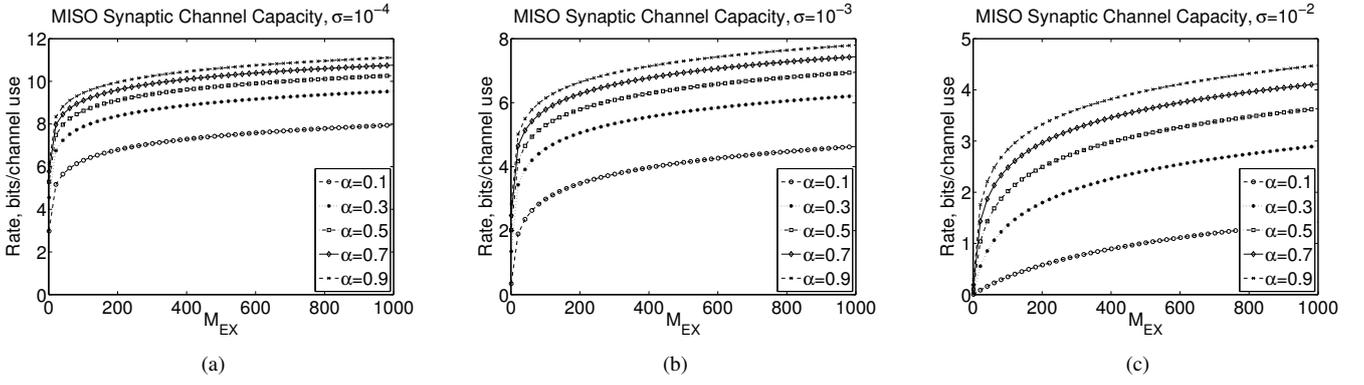


Fig. 3. MISO synaptic interference channel capacity for (a) $\sigma = 10^{-4}$, (b) $\sigma = 10^{-3}$, and (c) $\sigma = 10^{-2}$.

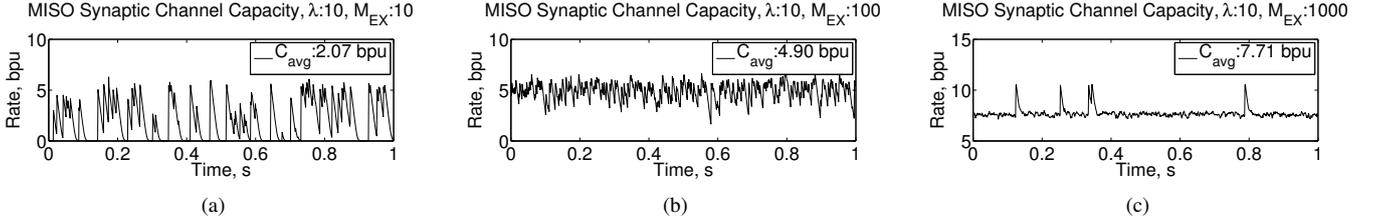


Fig. 4. MISO synaptic interference channel capacity for $\sigma^2 = 2.5 \times 10^{-8}$, (a) for $M_{EX} = 10$, (b) for $M_{EX} = 100$, and (c) for $M_{EX} = 1000$.

communication channels. We show that synaptic transmission performance is enhanced as stronger connection strengths are established. Furthermore, in comparison with maximum achievable rates in neuronal synaptic interference channel, we show that actual adaptive synaptic weight update mechanism performs close to the best achievable characteristics, and also enables high communication performance in terms of postsynaptic rate. Open issues include the effect of incorporation of correlation among the synaptic channel inputs on the postsynaptic throughput, and detailed analysis of interference among multi-terminal neuronal connections.

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Algorithm 1 Modification of the Synaptic Weight

Require: $t_{pre}(1) = 0, t_{post}(1) = 0$
Ensure: $T = 1000\text{ms}, \Delta t = 0.01\text{ms}$

- 1: **for** $i = 1, i++$, while $i < \frac{T}{\Delta t}$ **do**
- 2: $t_{pre}(i+1) - t_{pre}(i) \sim \text{Exp}(1/\lambda)$
- 3: $t_{post}(i+1) - t_{post}(i) \sim \text{Exp}(1/\lambda) * f_{\Delta t}(\delta t) * f_{\Delta t}(\delta t)$
- 4: **end for**
- 5: **for** $k = 1, k++$, while $k < \frac{T}{\Delta t}$ **do**
- 6: $t = t + \Delta t$
- 7: $M(t) = \exp(-t/\tau_+)$
- 8: $P_a(t) = \exp(-t/\tau_+)$
- 9: **if** $t_{pre}(k) = t$ **then** {Synapse **a** receives an AP}
- 10: $t_{pre} \leftarrow t$
- 11: **if** AP=1 **then** {Presynaptic AP is excitatory}
- 12: $g_{ex} \leftarrow g_{ex} + \bar{g}_a$
- 13: **else** AP=-1 {Presynaptic AP is inhibitory}
- 14: $g_{in} \leftarrow g_{in} + \bar{g}_{in}$
- 15: **end if**
- 16: $P_a(t) \leftarrow P_a(t) + A_+$
- 17: $\bar{g}_a \leftarrow \bar{g}_a + M(t)\bar{g}_{max}$
- 18: **if** $\bar{g}_a < 0$ **then**
- 19: $\bar{g}_a \leftarrow 0$
- 20: **end if**
- 21: **if** $t_{post}(k) = t$ **then** {Postsynaptic neuron fires an AP}
- 22: $t_{post} \leftarrow t$
- 23: $\delta t \leftarrow t_{pre} - t_{post}$
- 24: **if** $\delta t < 0$ **then**
- 25: $g(\delta t) = A_+ \exp(\delta t/\tau_+)$
- 26: **else** $\delta t \geq 0$
- 27: $g(\delta t) = -A_- \exp(-\delta t/\tau_-)$
- 28: **end if**
- 29: $\bar{g}_{max} \leftarrow \bar{g}_{max} + \bar{g}_{max}g(\delta t)$
- 30: $M(t) \leftarrow M(t) - A_-$
- 31: $\bar{g}_a \leftarrow \bar{g}_a + P_a(t)\bar{g}_{max}$
- 32: **if** $\bar{g}_a > \bar{g}_{max}$ **then**
- 33: $\bar{g}_a \leftarrow \bar{g}_{max}$
- 34: **end if**
- 35: **end if**
- 36: **else** $t_{pre}(k) \neq t$ {Synapse **a** does not receive an AP}
- 37: $g_{ex} = \exp(-t/\tau_{EX})$
- 38: $g_{in} = \exp(-t/\tau_{IN})$
- 39: **end if**
- 40: **end for**
