On Molecular Multiple-Access, Broadcast, and Relay Channels in Nanonetworks

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ABSTRACT

Molecular communication is a novel paradigm that uses molecules as an information carrier to enable nanomachines to communicate with each other. Interconnections of the nanomachines with molecular communication is envisioned as a nanonetwork. Nanonetworks are expected to enable nanomechines to cooperatively share information such as odor, flavour, light, or any chemical state. In this paper, we develop and present models for the molecular multiple-access, broadcast, and relay channels in a nanonetwork and derive their capacity expressions. Numerical results reveal that the molecular multiple-access of nanomachines to a single nanomachine can be possible with the high molecular communication capacity by selecting the appropriate molecular communication parameters. Similarly, the molecular broadcast can also allow a single nanomachine to communicate with a number of nanomachines with high molecular communication capacity. As a combination of the molecular multiple-access and broadcast channel, we show that the molecular relay channel can improve the molecular communication capacity between two nanomachines using a relay nanomachine.

Keywords

Molecular communication, Nanonetworks, Molecular multiple-access, broadcast, and relay channels.

1. INTRODUCTION

Molecular communication enables nanomachines to communicate with each other using molecules as a communication carrier [1]. A number of nanomachines communicating with each other using molecular communication is envisioned as a nanonetwork. Nanonetworks allow nanomechines to cooperatively share molecular information to achieve a specific task from nuclear, biological and chemical defense to food and water quality control [2].

Bionetics '08, November 25-28, 2008, Hyogo, Japan

In a traditional communication network with many senders and many receivers communicating with each other, there are mainly three kinds of communication channels called as multiple-access, broadcast, and relay channels. Similarly, in a nanonetwork with many Transmitter Nanomachines (TNs) and many Receiver Nanomachines (RNs) communicating with each other, we define three kinds of molecular channels called as molecular multiple-access channel, molecular broadcast channel, and molecular relay channel as follows:

- *Molecular multiple-access channel* is a molecular communication channel in which multiple TNs transmit molecular information to a single RN.
- *Molecular broadcast channel* is a molecular communication channel in which single TN transmits the molecular information to multiple RNs.
- *Molecular relay channel* is a molecular communication channel in which single TN transmits the molecular information to an RN using at least one nanomachine as a relay node.

There exist several research efforts on the molecular communication in the literature. In [1], research challenges in molecular communication is manifested. In [3], the concept of molecular communication is introduced and the first attempt for design of molecular communication system is performed. In [4], a molecular motor communication system for molecular communication is introduced. In [5], an autonomous molecular propagation system is proposed to transport information molecules using DNA hybridization and biomolecular linear motors. In [2], a survey on nanonetworking with molecular communication is introduced. In our previous work [6], we introduced an information theoretical approach for molecular communication, derived a closed-form expression for a single molecular communication channel capacity between two nanomachines and proposed an adaptive error compensation technique for molecular communication. The existing studies on the molecular communication include feasibility of the molecular communication and design schemes for molecular communication system. However, none of these studies investigate a nanonetwork to find out the capacity of a molecular channel between two arbitrary nanomachines. Moreover, it is imperative to investigate capacity of molecular communication among multiple nanomachines to develop efficient molecular communication strategies for nanonetworks. In this paper, using the single molecular channel model proposed in our previous work [6],

we model the molecular multiple-access, broadcast and relay channels and derive capacity expressions for these molecular channels.

The remainder of this paper is organized as follows. In Section 2, we briefly introduce an review single molecular communication channel model and its capacity. Based on the single molecular communication channel, we model molecular multiple-access, broadcast, and relay channels and derive their capacity expressions in Section 3, 4, and 5, respectively. In Section 6, we provide the numerical results and we give concluding remarks in Section 7.

2. SINGLE MOLECULAR COMMUNICATION CHANNEL

In this section, we briefly review a single molecular channel model and its capacity expression derived in our previous work [6]. The single molecular channel model is used for modeling of molecular multiple-access, broadcast and relay channels and to derive their capacity expressions in the following.

In the single molecular channel, we assume that Transmitter Nanomachine (TN) emits one kind of molecule called A with a time-varying concentration of $L(t)$ according to the following emission pattern [7] which is similar to alternating square pulse, i.e.,

$$L(t) = \begin{cases} L_{ex}, & \text{with probability } P_A \text{ in } jt_H \leq t \leq (j+1)t_H \\ 0, & \text{with probability } (1-P_A) \quad \text{otherwise} \end{cases} \tag{1}$$

where $j = (0, 1, ...)$, t_H is the duration of a pulse, L_{ex} is concentration of molecules A emitted by TN and P_A is the probability that TN emits molecules A with the concentration L_{ex} during t_H. Furthermore, we assume that RN has N receptors called R on its surface. The receptors enable RN to receive the molecules which bind to their surface. When TN emits molecules A with concentration L_{ex} during t_H, some of molecules bind to these receptors and these bound molecules generate a concentration in RN.

Here, similar to the traditional digital communication having two bits called logic 0 and logic 1, we assume two molecular communication bits called molecular bit A and molecular bit 0. To transmit molecular bit A, TN emits molecules A with concentration L_{ex} during t_H. For transmission of molecular bit 0, TN emits no molecule to its surrounding environment during t_H.

At RN side, these bits are inferred via concentration of molecules A. If RN receives a concentration of molecules A greater than a prescribed concentration S ($\mu mol/liter$), the RN decides that the TN transmitted molecular bit A. Conversely, if the RN receives molecules A with a concentration less than S, the RN decides that the TN transmitted molecular bit 0.

If TN emits molecules A during t_H with concentration L_{ex}, expected concentration of delivered molecules A, i.e., NA, can be given as [6]

$$NA = \int_0^{t_H} \frac{k_1 L_{ex} N}{k_{-1} + k_1 L_{ex}} (1 - e^{-t(k_{-1}+k_1 L_{ex})}) dt. \tag{2}$$

where k_1 ($\mu mol/liter/sec.$) and k_{-1} ($1/sec.$) are the binding and release rates, respectively, N ($\mu mol/liter$) is the concentration of receptors (R) on RN.

Since the molecular diffusion continues after every t_H interval, the previous molecular bits can be received in the current interval by RN. Therefore, the number of delivered molecules A in a given interval also depends on molecule concentration emitted in the previous intervals. Here, we assume that only the last molecule concentration affects the current molecular transmission since the number of delivered molecules exponentially decay after t_H seconds. Hence, the effect of the last emitted molecule concentration on the current molecule emission, i.e., NP can be given as follows

$$NP = P_A NA \int_0^{t_H} e^{(-k_{-1}t)} dt \tag{3}$$

Using (2), for the case that TN emits A during t_H, expected concentration of delivered molecules A, i.e., $E[S_A]$, can be given as

$$E[S_A] = NA + NP. \tag{4}$$

where we assume that S_A is normally distributed random variable with the distribution $N(E[S_A], \sigma_A^2)$. Many events in nature can be approximated with the normal distribution corresponding to central limit theorem. However, the normal distribution assumption for S_A is not a strict assumption. Instead of the normal distribution, it can be easily considered as an any other distribution such as Poisson or exponential. Therefore, the normal distribution assumption for S_A is reasonable to effectively investigate the molecular channel capacity. Since S_A cannot be negative, the minimum value of S_A is equal to 0. In any normal distribution, % 99.7 of the observations fall within 3 standard deviations of the mean. Therefore, $E[S_A] - 3\sigma_A = 0$ can be given, that is, $\sigma_A = E[S_A]/3$ for the distribution of S_A and $\mu_A = E[S_A]$ and $\sigma_A = (E[S_A]/3)$.

For the case TN emits no molecules during t_H, the number of delivered molecules A only depends on lastly emitted molecule concentration. Therefore, following (3), the expected value of delivered molecules A within t_H for the transmission of molecular bit 0, i.e., $E[S_0]$, is given by

$$E[S_0] = NP \tag{5}$$

where similar to S_A, S_0 has the distribution $N(\mu_0, \sigma_0^2)$, where σ_0 can be given as $\sigma_0 = \mu_0/3$ and $\mu_0 = E[S_0]$.

For the molecular communication between TN and RN, two molecular bits are available. Every time when TN transmits a molecular bit, concentration of delivered molecules determines the success of the transmission. If TN transmits molecular bit A, at least S number of molecules[1] A must be delivered to RN within time interval t_H for a successful delivery of a molecular bit A. If TN transmits molecular bit 0, number of molecules A delivered within t_H must be less than S for a successful delivery of molecular bit 0.

If RN receives at least S number of molecules A, it infers that TN emitted the molecular bit A. Thus, we obtain a maximum bound for the probability p_1 that TN achieves to deliver molecular bit A as follows

[1]Since concentration of molecules ($\mu mol/liter$) can be converted to number of molecules by multiplying Avagadro constant (6.02×10^{23}), we interchangeably use the number of molecules for the concentration of molecules.

$$p_1(S_A \geq S) = \int_S^{\infty} \frac{1}{\sigma_A 2\pi} e^{-\frac{(x-\mu_A)^2}{\sigma_A^2}} dx \quad (6)$$

Hence, TN achieves to deliver molecular bit A with maximum probability p_1 and RN receives molecular bit 0 instead of the molecular bit A such that TN does not succeed to deliver A with probability $(1 - p_1)$.

For the successful delivery of a molecular bit 0, TN must deliver a number of molecules A that is less than S to RN ($S_0 \leq S$). Therefore, the maximum bound for probability p_2 that TN achieves to deliver molecular bit 0 is given by

$$p_2(S_0 \leq S) = \int_0^{S} \frac{1}{\sigma_0 2\pi} e^{-\frac{(x-\mu_0)^2}{\sigma_0^2}} dx \quad (7)$$

Hence, for the transmission of molecular bit 0, TN achieves to deliver molecular bit 0 with maximum probability p_2 and it does not achieve to deliver molecular bit 0, instead, it incorrectly delivers molecular bit A with probability $(1 - p_2)$.

According to P_A, p_1, and p_2, we can model the molecular channel between TN and RN as a symmetric channel. If we consider that TN emits molecular bit X and RN receives molecular bit Y, then the transition matrix of the molecular channel can be given as follows

$$P(Y/X) = \begin{pmatrix} p_1 P_A & (1-p_2)(1-P_A) \\ (1-p_1)P_A & p_2(1-P_A) \end{pmatrix}$$

Based on the transition matrix $P(Y/X)$, the mutual information between X and Y which states the number of distinguishable molecular bits, i.e., $I(X;Y)$, can be given as follows

$$I(X;Y) = \left(H\left(p_1 P_A + (1-p_2)(1-P_A), (1-p_1)P_A + p_2(1-P_A) \right) \right) - \left(P_A H(p_1, 1-p_1) + (1-P_A) H(p_2, 1-p_2) \right) \quad (8)$$

where $H(.)$ denotes the entropy. Using (8), the capacity of the single molecular channel between TN and RN i.e., SC, can be expressed as

$$SC = max(I(X;Y)). \quad (9)$$

Next, using the single molecular communication channel model, assumptions and notations presented above, we model the molecular multiple access, broadcast, and relay channels and derive their capacity expressions.

3. MOLECULAR MULTIPLE - ACCESS CHANNEL

In the molecular multiple-access channel, multiple TNs communicate with a single RN. Here, we assume that number of n TNs ($TN_1 ... TN_n$) communicate with a single RN as shown in Fig. 1. We also assume that each nanomachine has a self-identifying label[2] and attaches this label to the emitted molecules. This mechanism provides a simple addressing scheme. Here, we also assume that TN_i transmits molecular bit A with probability P_{Ai} and concentration L_{ex} using the binding and release rates k_1^i and k_{-1}, respectively. Similar to the single molecular communication channel, if we assume that there is no contention among TNs to access the molecular multiple-access channel, using (2) and (3), the expected number of molecules delivered in transmission of molecular bit A by TN_i, i.e., $E[S_A^i]$, can be computed as

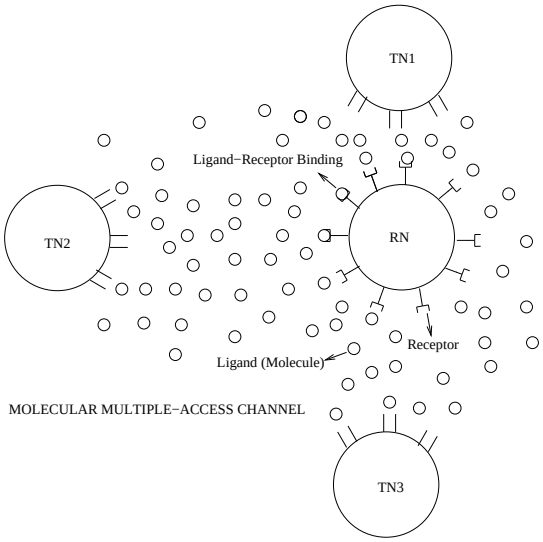


Figure 1: Molecular multiple-access channel with three transmitter nanomachines and one receiver nanomachine.

$$E[S_A^i] = NA + P_{Ai} NA \int_0^{t_H} e^{(-k_{-1}t)} dt \quad (10)$$

where NA can be computed using (2) with k_1^i, k_{-1}, and L_{ex}. Here, similar to S_A, S_A^i has the distribution $N(\mu_{Ai}, \sigma_{Ai}^2)$, where $\mu_{Ai} = E[S_A^i]$ and $\sigma_{Ai} = E[S_A^i]/3$.

In transmission of molecular bit 0, using (3), the expected number of molecules delivered by TN_i, i.e., $E[S_0^i]$ can be expressed as

$$E[S_0^i] = P_{Ai} NA \int_0^{t_H} e^{(-k_{-1}t)} dt \quad (11)$$

where similar to S_0, ${S_0}^i$ has the distribution $N(\mu_{0i}, \sigma_{0i}^2)$ such that $\mu_{0i} = E[S_0^i]$ and $\sigma_{0i} = E[S_0^i]/3$.

S_A^i and S_0^i are the concentrations of molecules delivered by TN_i in transmission of molecular bit A and 0 similar to the single transmitter case introduced in Section 2. Since there are n nanomachines contending in the multiple-access channel for a single type of receptor on RN, concentration of delivered molecules for each nanomachine is reduced with respect to the single transmitter case. For this molecular multiple-access channel, molecule concentration delivered by TN_i in transmission of molecular bit A, i.e., M_A^i, can be expressed as

$$M_A^i = K S_A^i \quad (12)$$

[2]Molecule labeling is the most popular experimental method to investigate the ligand-receptor interactions [8] and there are mainly three kinds of labeling process called as radio, enzymatic, and fluorescent labeling to detect the ligand-receptor binding [9]. Here, we assume that each nanomachine has self-identifying labeled molecules to be emitted.

where K is a constant reducing factor. In [10], a model is proposed to find concentration of bound molecules (delivered molecules) for the case in which different molecules bind to a single kind of receptors with a constant concentration. Here, using the model introduced in [10], K can be expressed as

$$K = \frac{N}{N + \sum_{j \neq i}^{n} \Big(P_{Aj} E[S_A^j] + (1 - P_{Aj}) E[S_0^j] \Big)} \tag{13}$$

where N ($\mu mol/liter$) is the receptor concentration on RN, $\sum_{j \neq i}^{n} \Big(P_{Aj} E[S_A^j] + (1 - P_{Aj}) E[S_0^j] \Big)$ denotes the average molecule concentration delivered by other TNs contending on the molecular multiple-access channel. Since K is a constant, M_A^i has the distribution $N(K\mu_{Ai}, (K\sigma_{Ai})^2)$.

Similarly, in transmission of molecular bit 0, molecule concentration delivered by TN_i, i.e., M_0^i can be given as

$$M_0^i = K S_0^i \tag{14}$$

where M_0^i has the distribution $N(K\mu_{0i}, (K\sigma_{0i})^2)$ since S_0^i is a normally distributed random variable and K is a constant.

In the molecular multiple-access channel, for the successful delivery of molecular bit A, TN_i must deliver at least S number of molecules to RN. The maximum bound for probability p_{1i} that TN_i achieves to deliver molecular bit A is given by

$$p_{1i}(M_A^i \geq S) = \int_S^{\infty} \frac{1}{K\sigma_{Ai}2\pi} e^{-\frac{(x - K\mu_{Ai})^2}{(K\sigma_{Ai})^2}} dx \tag{15}$$

Hence, TN_i achieves to deliver molecular bit A with maximum probability p_{1i} and fails to deliver molecular bit A with probability $(1 - p_{1i})$.

For the successful delivery of molecular bit 0, TN_i must deliver at most S number of molecules to RN. Therefore, the maximum bound for probability p_{2i} that TN_i achieves to deliver molecular bit 0 can be given as

$$p_{2i}(M_0^i \leq S) = \int_0^{S} \frac{1}{K\sigma_{0i}2\pi} e^{-\frac{(x - K\mu_{0i})^2}{(K\sigma_{0i})^2}} dx \tag{16}$$

Hence, TN_i achieves to deliver molecular bit 0 with maximum probability p_{2i} and fails to deliver with probability $(1 - p_{2i})$.

According to P_{Ai}, p_{1i}, and p_{2i}, we can model the molecular channel between TN_i and RN similar to a symmetric channel. If we consider that TN_i emits molecular bit X and RN receives molecular bit Y, the mutual information between X and Y, i.e., $I^i(X;Y)$, can be computed using p_{1i}, p_{2i}, and P_{Ai} by (8).

Based on $I^i(X;Y)$, the capacity of the molecular channel between TN_i and RN, i.e., MC_i, can be expressed as

$$MC_i = max(I^i(X;Y)) \tag{17}$$

Hence, capacity of the molecular multiple-access channel, i.e., MC, can be given as follows

$$MC = max\Big(\sum_{i=1}^{n} I^i(X;Y) \Big) \tag{18}$$

4. MOLECULAR BROADCAST CHANNEL

In the molecular broadcast channel, single TN communicates with multiple RNs as shown in Fig. 2. Here, we assume that a single TN communicates with number of n RNs ($\mathrm{RN}_1 ... \mathrm{RN}_n$). We also assume that TN attaches its label on the molecules to enable RNs to infer which nanomachine transmits its molecules to them. In the molecular broadcast channel, we assume that the molecules emitted by TN uniformly diffuse to all direction in the surrounding environment. Therefore, each RN receives a molecule concentration independent of other RNs in the channel such that RNs do not interfere with each other. Therefore, TN delivers different number of molecules to each RN according to their binding (k_1) and release (k_{-1}) rates which are considerably affected from the locations of RNs with respect to TN. Here, we assume that TN transmits molecular bit A to RN_i with probability P_A using binding rate k_1^i and release rate k_{-1}.

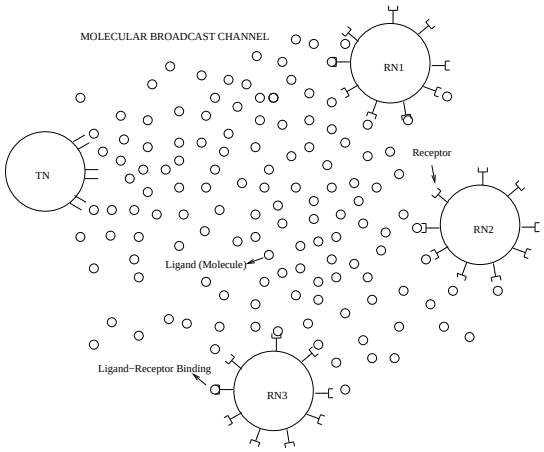


Figure 2: Molecular broadcast channel with one transmitter nanomachine and three receiver nanomachines.

Hence, the molecular channel between TN and any RN_i has the same molecule delivery capability with the single molecular channel such that RN_i can independently receive any molecule concentration according to its binding rate (k_1^i) and release rate (k_{-1}). In the molecular broadcast channel, using k_1^i and k_{-1}, the capacity of the molecular channel between TN and RN_i, i.e., BC_i, can be directly found using the mutual information of single molecular channel ($I^i(X;Y)$) given in (6), (7), and (8) as follows

$$BC_i = max\big(I^i(X;Y)\big) \tag{19}$$

Hence, the total capacity achieved in the broadcast channel from TN to n number of RNs, i.e., BC, can be given as $BC = \sum_{i=1}^{n} BC_i$.

5. MOLECULAR RELAY CHANNEL

In the molecular relay channel, a single TN transmits molecular information to RN using at least one nanomachine as relay node as shown in Fig. 3. Here, we assume that there is one nanomachine denoted by H as a relay node such that it has the capability of molecule emission and reception[3]. This way, it can receive the molecular information

[3]In nature, many biological entities have the both of

from TN and forward the received molecular information to RN. Similar to RN, H has the receptors on its surface with the concentration N ($\mu mol/liter$) and it also has the molecule emission capability with the emission pattern given in (1). We also assume TN attaches its self-identifying label to emitted molecules and H also attaches the label of TN and its label to molecules emitted by it. This enables RN to inform that H helps the molecular communication between TN and RN. In addition to this labeling process, we also assume that H foreknows next molecular bit, which will be emitted by TN, to help the molecular communication between TN and RN. Using this information provided by TN, H emits the same molecular bit with TN in each transmission interval t_H. This can be interpreted as an encoding process performed in the traditional relay channel to help the communication between source and destination nodes.

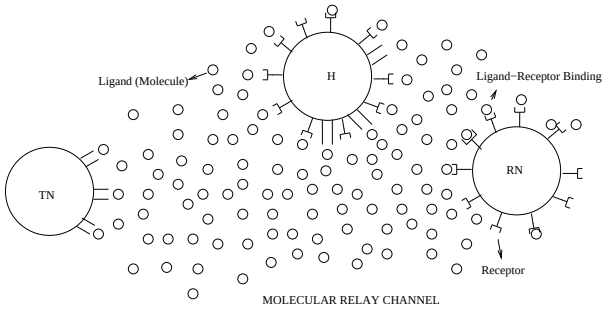


Figure 3: Molecular relay channel between transmitter and receiver nanomachine with one relay nanomachine.

Similar to the traditional relay channel with one multiple-access and one broadcast channel, the molecular relay channel also consists of one molecular broadcast channel and one molecular multiple-access channel. In the broadcast channel, TN transmits the molecular information to H and RN. In this channel, the capacities from TN to H and RN, i.e., BC_h and BC_r, respectively, can be computed using (2)-(9) as introduced in Section 4. In the multiple-access channel, H and TN transmit the molecular information to RN. In this channel, we denote the capacity between H and RN as MC_h and we denote the capacity between TN and RN as MC_r.

Traditionally, the max-flow min-cut theorem [12] is the most popular theorem providing a satisfactory solution for the capacity of simple relay channel with a single relay node [11]. Similarly, in the molecular relay channel, we adopt the max-flow min-cut theorem for the capacity as follows.

According to the max-flow min-cut theorem [12], the molecular relay channel with a single relay node H has two cut sets. First cut set includes (TN,H) and (TN,RN) and second includes (H,RN) and (TN,RN). The first cut set includes the molecular broadcast channel from TN to H and RN. The second cut set includes the molecular multiple-access channel from TN and H to RN. Therefore, the capacity of the molecular relay channel, i.e., RC, is equal to the minimum capacity of these cut sets [12] and can be given as

$$RC = min\big(max(BC_h, BC_r), \overline{MC}\big) \tag{20}$$

where $\overline{MC}$ is the capacity of the molecular multiple-access channel from TN and H to RN. Although H and TN emits the same molecular bit to RN in each transmission interval t_H, H and TN also contend on the receptors of RN to deliver their molecules having the same label to the single type of receptors (R) on RN. Therefore, in this molecular multiple-access channel, using the method introduced in Section 3 the molecular communication capacities MC_h, MC_r, and $\overline{MC}$ are derived as follows.

We assume that in the multiple-access channel TN and H have the binding rate k_1^{TN} and k_1^H, respectively and have the same release rate k_{-1} and they have the same molecular bit transmission probability P_A. If we assume that TN and H emits molecular bit A and do not contend as in a single molecular communication channel, expected concentration of molecules delivered to RN by TN and H, i.e., $E[S_A^{TN}]$ and $E[S_A^H]$, can be computed using (2), (3) and (4). However, in the multiple access channel, concentration of molecules delivered by TN and H, i.e., M_A^{TN} and M_A^H, can reduce due to the contention. Therefore, the reduced concentration of molecules delivered by TN and H, i.e., M_A^{TN} and M_A^H, can be given as follows

$$M_A^{TN} = K_{ATN} S_A^{TN}, \quad M_A^H = K_{AH} S_A^H \tag{21}$$

where K_{ATN} and K_{AH} are constant reducing factors, N ($\mu mol/liter$) is the concentration of receptors on RN. Using the concept given in [10], these reducing factors can be given as follows

$$K_{ATN} = \frac{N}{N + E[S_A^H]}, \quad K_{AH} = \frac{N}{N + E[S_A^{TN}]} \tag{22}$$

where the reducing factors K_{ATN} and K_{AH} are slightly different from the reducing factor (K) given in (13), because H and TN emits the same molecular bit in each interval t_H.

In transmission of molecular bit 0, expected number of molecules delivered to RN by TN and H, i.e., $E[M_0^{TN}]$ and $E[M_0^H]$, can be given as follows

$$E[M_0^{TN}] = K_{0TN} E[S_0^{TN}], \quad E[M_0^H] = K_{0H} E[S_0^H] \tag{23}$$

where $E[S_0^{TN}]$ and $E[S_0^H]$ are the expected number of molecules delivered by TN and H without the contention on RN receptors. $E[S_0^{TN}]$ and $E[S_0^H]$ can be computed using (2), (3) and (5). K_{0TN} and K_{0H} are constant reducing factors and can be given as

$$K_{0TN} = \frac{N}{N + E[S_0^H]}, \quad K_{0H} = \frac{E[S_0^H]N}{N + E[S_0^{TN}]}. \tag{24}$$

In the molecular multiple-access channel, if TN and H emits the molecular bit A, they must deliver at least S number of molecules to RN for the successful delivery of molecular bit A, that is, $M_A^{TN} + M_A^H \geq S$ must be satisfied. The maximum bound for probability p_1 that TN and H deliver molecular bit A to RN is given by

molecule emission and reception capabilities. Our assumption is based on this fact. Beyond this assumption, we do not consider the feasibility of these capabilities in a nanomachine.

$$p_1(M_A^{TN} + M_A^H \geq S) = \int_S^{\infty} \frac{e^{-\frac{(x-(K_{ATN}+K_{AH})\mu_A)^2}{((K_{ATN}+K_{AH})\sigma_A)^2}}}{(K_{ATN}+K_{AH})\sigma_A 2\pi} dx \quad (25)$$

where $M_A^{TN} + M_A^H$ is a normally distributed random variable, because S_A^{TN} and S_A^H have normal distribution as introduced in Section 3. Therefore, $(K_{ATN} + K_{AH})\mu_A$ and $((K_{ATN} + K_{AH})\sigma_A)^2$ are the mean and variance of $M_A^{TN} + M_A^H$, respectively, where μ_A and σ_A are the mean and variance of random variables S_A^{TN} and S_A^H similar to the single molecular channel in Section 2.

Hence, TN and H achieve to deliver molecular bit A with maximum probability p_1 and fail to deliver molecular bit A with probability $(1 - p_1)$.

For the successful delivery of molecular bit 0, TN and H must deliver at most S number of molecules to RN. Therefore, the maximum bound for probability p_2 that TN and H achieve to deliver molecular bit 0 can be given as

$$p_2(M_0^{TN} + M_0^H \leq S) = \int_0^{S} \frac{e^{-\frac{(x-(K_{0TN}+K_{0H})\mu_0)^2}{((K_{0TN}+K_{0H})\sigma_0)^2}}}{(K_{0TN}+K_{0H})\sigma_0 2\pi} dx \quad (26)$$

where $(K_{0TN} + K_{0H})\mu_0$ and $((K_{0TN} + K_{0H})\sigma_0)^2$ are the mean and variance of $M_0^{TN} + M_0^H$. Here, μ_0 and σ_0 are the mean and variance of random variables S_0^{TN} and S_0^H similar to the single molecular channel in Section 2.

Hence, TN and H achieve to deliver molecular bit 0 with maximum probability p_2 and fail to deliver molecular bit 0 with probability $(1-p_2)$. Similar to the symmetric channel, if we consider that TN and H emit molecular bit X and RN receives molecular bit Y, the mutual information between X and Y, i.e., $I^{mc}(X;Y)$, can be computed using p_1, p_2, and P_A by (8). Then, the molecular communication capacity for the multiple-access channel from TN and H to RN, i.e., $\overline{MC}$, can be obtained by maximizing $I^{mc}(X;Y)$. Hence, Using BC_h, BC_r, and $\overline{MC}$, the capacity of the molecular relay channel can be computed using (20).

6. NUMERICAL ANALYSIS

In this section, we present the numerical analysis on the molecular multiple-access, broadcast and relay channels. The aim of this analysis is to determine the molecular channel characteristics in multiple-access, broadcast, and relay cases. We also aim to observe the changes in these characteristics according to the molecular communication parameters such as number of nanomachines contending on the molecular channels, receptor concentration R, and threshold concentration S. We perform the numerical analysis using Matlab. We assume that TN and RN are randomly positioned in an environment, which may have different diffusion coefficients such that it allows TN to achieve different binding rates (k_1). We also assume that k_1 varies with distance (α) between TN and RN such that k_1 is inversely proportional with α ($k_1 \propto 1/\alpha$). Moreover, we assume that k_{-1} depends only on the properties of RN receptors and cannot be changed. The simulation parameters of the analysis are given in Table 1.

6.1 Molecular Multiple-Access Channel

We first observe the effect of the number of TNs (n), transmitting the molecular information to a single RN, on the capacity of the molecular multiple-access channel capacities

Table 1: Simulation Parameters

Binding rate (k_1)	0.1-0.3 $(\mu mol/liter/s)$
Release rate (k_{-1})	0.08 (s^{-1})
Distance between nanomachines (α)	$5^{-10} - 4 \times 10^{-9} m$
Number of nanomachines (n)	$1 - 20$
Concentration of molecules A (L_{ex})	1-4 $(\mu mol/liter)$
Duration of the pulses (t_H)	1 s
$\boldsymbol{N}$ $(\mu mol/liter)$	$0.5 - 3 \times 10^{-3}$
$\boldsymbol{S}$ $(\mu mol/liter)$	$1 - 7 \times 10^{-5}$

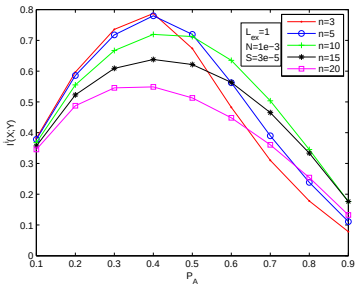
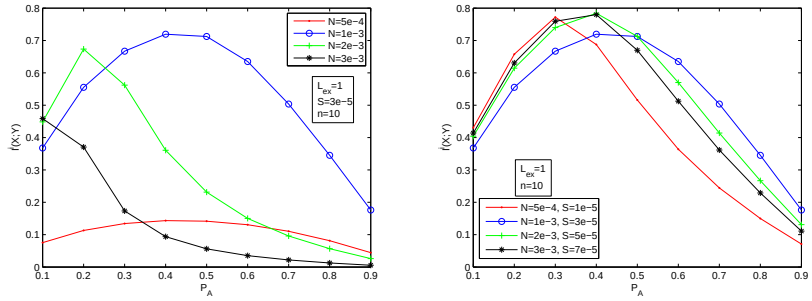


Figure 4: $I^i(X;Y)$ with varying P_A for different n.

MC_i given in (17). We assume that a number of TNs are located around the RN and all of them have the same binding rate k_1 ($k_1^i = k_1$), the same release rate k_{-1}, and the same molecular bit transmission probability P_A ($P_{Ai} = P_A$).

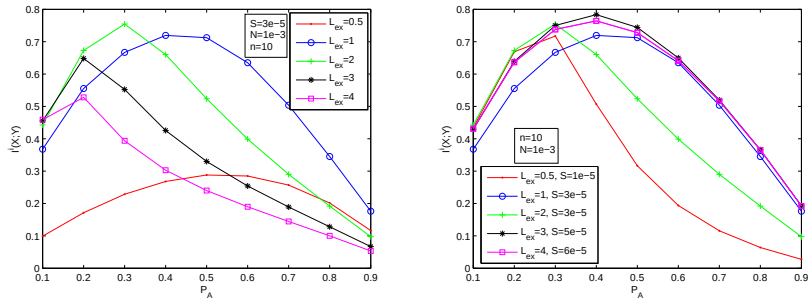
In Fig. 4, the mutual information achieved by TN_i is shown with varying P_A for different n. The molecule concentration delivered by TNs slightly increases as n increases. However, for the smaller n case, the delivered molecule concentration increases as P_A increases due to lower contention and in this case, erroneous molecular bit 0 mostly arises and $I^i(X;Y)$ decreases. Hence, as n decreases, smaller P_A values should be selected for providing higher molecular communication capacities in the molecular multiple-access channel.

In Fig. 5.a, we show the effect of different receptor concentration (N) on the mutual information achieved by TN_i ($I^i(X;Y)$) with the varying molecular bit transmission probability (P_A). As N increases, molecule concentration delivered to RN by each TN increases. For the smallest value of N ($N = 5 \times 10^{-4}$), every TN cannot deliver sufficient concentration, that is greater than S, to achieve to deliver molecular bit A and the probability of error in transmission of molecular bit A increases. For $= 1 \times 10^{-3}$, molecular bits A and 0 can be satisfactorily delivered by TNs and $I^i(X;Y)$ increases. However, as N further increases, $I^i(X;Y)$ decreases. This is because excessively delivered molecule concentration with increasing N results in erroneous molecular bit 0 and $I^i(X;Y)$ decreases. Since the successful delivery of the molecular bits is considerably affected by the selected threshold concentration S, the selection of S is critical to achieve higher molecular communication capacity. In Fig. 5.b, $I^i(X;Y)$ is shown with varying P_A for different N and S. Since the increasing N results in higher molecule concentration delivered by TNs, we increase S corresponding to the increase in N. Contrary to Fig. 5.a, by regulating S according to the increasing N, it can be possible to achieve higher molecular communication capacity.



(a) (b)

Figure 5: (a) $I^i(X;Y)$ with the varying P_A for different N. (b) $I^i(X;Y)$ with varying P_A for different N and S.



(a) (b)

Figure 6: (a) $I^i(X;Y)$ with P_A for different L_{ex}. (b) $I^i(X;Y)$ with varying P_A for different L_{ex} and S.

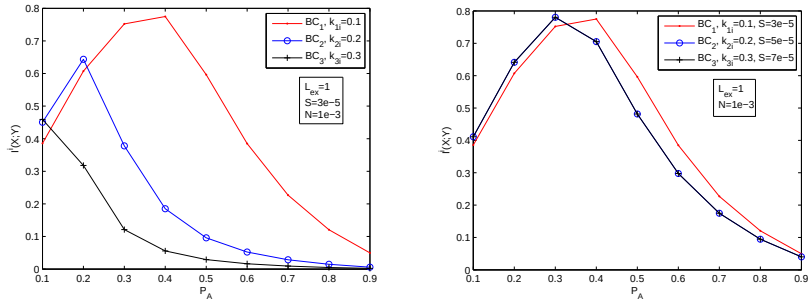
In Fig. 6.a, we show the effect of different concentration of emitted molecules (L_{ex}) on the mutual information achieved by TN_i ($I^i(X;Y)$) with varying molecular bit transmission probability (P_A). Similar to the effect of N, as L_{ex} increases, delivered molecule concentration increases. For the smallest L_{ex} ($L_{ex} = 0.5$), $I^i(X;Y)$ is very low since the sufficient concentration greater than S for molecular bit A cannot be delivered. However, the appropriate L_{ex} can be selected to achieve higher molecular communication capacity as shown in Fig. 6.a. In addition, according to L_{ex}, S can also be regulated for the higher capacity. In Fig. 6.b, $I^i(X;Y)$ is shown with varying P_A for different L_{ex} and S. Here, we increase S corresponding to the increasing L_{ex}. As shown in Fig. 6.b, the appropriate selection of S according to L_{ex} enables TNs to achieve the higher molecular communication capacities.

6.2 Molecular Broadcast Channel

In the molecular broadcast channel, we assume that single TN transmits to three RNs called as RN_1, RN_2, and RN_3 and these RNs achieve the corresponding molecular communication capacities BC_1, BC_2, and BC_3. We also assume that each RN has different binding rate (k_1^i) according to its physical location and they have the same release rate k_{-1}. As introduced in Section 4, similar to the single molecular communication channel, each RN can achieve different molecular communication capacity with respect to its binding and release rates since we assume that the emitted molecules uniformly diffuse to all directions in the environment. In Fig. 7.a, we show the mutual information ($I^i(X;Y)$) achieved by each RN in the molecular broadcast channel with varying molecular bit transmission probability (P_A). RN_1 with the smallest binding rate can achieve higher capacity than the others. The main reason for this is excessive molecule delivery in the higher binding rate cases such that the excessive molecule concentration received by RN_2 and RN_3 results in delivery of erroneous molecular bit 0 as P_A increases. However, S can be regulated for the higher molecular communication capacity. As shown in Fig. 7.b, by regulating S according to the binding rates, it is possible to achieve higher molecular communication capacities.

6.3 Molecular Relay Channel

In the molecular relay channel, a single relay nanomachine H helps the molecular communication between TN and RN. Here, we assume that H has higher binding rate than RN. Since H is closer to TN, it is reasonable for H to have higher binding rate and to deliver more molecule concentration than RN. In Fig. 8, the capacity of the relay channel RC is shown with and without relay node H. For smaller P_A values, the relay node H can improve the molecular communication capacity between TN and RN as P_A increases. However, as P_A further increases, H cannot improve the capacity. This is because the increasing P_A results in excessive molecule delivery in the transmission of molecular bit 0 and the erroneous molecular bits 0 mostly arise. Therefore, the capacity is reduced by the erroneous molecular bit 0. Hence, P_A should be appropriately selected to improve the molecular communication capacity using a relay node. For example, in this case given in Fig. 8 P_A



(a) (b)

Figure 7: (a) $I^i(X;Y)$ with varying P_A. (b) $I^i(X;Y)$ with varying P_A for different S.

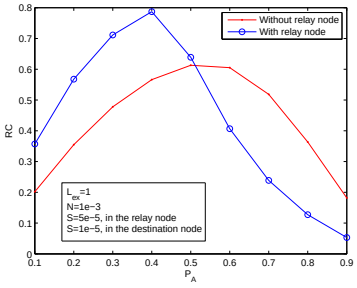


Figure 8: RC with and without the relay node H.

should be selected as a value smaller than 0.5 to improve the communication capacity between TN and RN using the relay node H.

7. CONCLUSION

In this paper, we introduce the molecular multiple-access, broadcast and relay channels and derive their capacity expressions. Theoretical and numerical results reveal that the molecular multiple-access of nanomachines to a single nanomachine can be possible with the high molecular communication capacity by selecting the appropriate molecular communication parameters. Similarly, the molecular broadcast can also allow a single nanomachine to communicate with a number of nanomachines with high molecular communication capacity by selecting the appropriate parameters. Combining the molecular multiple-access and broadcast channel, we show that a relay nanomachine can improve the capacity of molecular communication from a source na no mac hi ne to a destination nanomachines. Our ongoing works aim to develop molecular communication algorithms to enable arbitrary nanomachines in a nanonetwork to efficiently communicate with each other through the molecular multiple-access, broadcast and relay channels.

8. ACKNOWLEDGMENTS

This work was supported in part by Turkish Academy of Sciences under TUBA-GEBIP Award and by TUBITAK under the Grant #106E179.

9. REFERENCES

[1] S. Hiyama, Y. Moritani, T. Suda, R. Egashira, A. Enomoto, M. Moore and T. Nakano, "Molecular Communication", *In Proc. of NSTI Nanotech 2005*, Anaheim, California, USA.

[2] I. F. Akyildiz, F. Brunetti, C. Blazquez, "NanoNetworking: A New Communication Paradigm", *Computer Networks Journal (Elsevier)*, June 2008.

[3] T. Suda, M. Moore, T. Nakano, R. Egashira, A. Enomoto, "Exploratory Research on Molecular Communication between Nanomachines", *In Proc. of GECCO 2005*, June 25-29, 2005, Washington, DC, USA.

[4] M. Moore, A. Enomoto, T. Nakano, R. Egashira, T. Suda, A. Kayasuga, H. Kojima, H. Sakakibara, K. Oiwa, "A Design of a Molecular Communication System for Nanomachines Using Molecular Motors", *In Proc. of IEEE PERCOMW 2006*, Italy, 2006.

[5] S. Hiyama, Y. Isogawa, T. Suda, Y. Moritani, K. Sutoh, "A Design of an Autonomous Molecule Loading/Transporting/Unloading System Using DNA Hybridization and Biomolecular Linear Motors", *In Proc. of European Nano Systems 2005*, Paris, France, December 2005.

[6] B. Atakan, O. B. Akan, "On Channel Capacity and Error Compensation in Molecular Communication", to appear in *Springer Trans. on Computational System Biology*.

[7] V. Krivan, P. Lansky, J. P. Rospars, "Coding of periodic pulse stimulation in chemoreceptors", *Elsevier Biosystem*, vol. 67, pp. 121-128, 2002.

[8] D. A. Lauffenburger, J. Linderman, "Receptors, Models for Binding, Trafficking and Signaling", *Oxford University Press*, Oxford, 1993.

[9] M. A. Cooper, "Label-free screening of bio-molecular interactions", *Analytical and Bioanalytical Chemistry*, vol. 377, pp. 834-842, 2003.

[10] R. Malaka, T. Ragg, and M. Hammer, "Kinetic models of odor transduction implemented as artificial neural networks", *Biological Cybernetics*, vol. 73, pp. 195-207, 1995.

[11] T. M. Cover, J. A. Thomas, "Elements of information theory", *John Wiley-Sons*, 2006.

[12] L. R. Ford, D. R. Fulkerson, "Flows in Networks", *Princeton University Press*, 1974.